
DYNAMICS AND STABILITY OF MULTIPLICATIVE FIBONACCI-TYPE RECURRENCE

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ABSTRACT

An interesting variation of the Fibonacci sequence arises when each new term is defined as the product, rather than the sum, of the two preceding terms. In this work, a generalised multiplicative Fibonacci-type recurrence is introduced. For selected particular cases, closed-form expressions for the general term are derived. The recurrence is then reformulated as a two-dimensional discrete dynamical system, allowing for the analysis of the existence of fixed points and the study of their local stability. However, under appropriate positivity conditions, the nonlinear structure becomes considerably simpler after a logarithmic renormalisation, by which the multiplicative recurrence is transformed into an additive, Fibonacci-type one. Within this framework, an explicit representation for the renormalised dynamics is derived through a Fibonacci-type variation-of-constants formula. The asymptotic behaviour of three particular cases is included to illustrate the theoretical results. The proposed framework provides a unified perspective on multiplicative analogues of Fibonacci-type sequences and suggests further extensions to other nonlinear recurrences of similar structure.

Keywords Multiplicative Fibonacci sequence; perturbation; stability; recurrence relation.

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