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# DE MOIVRE'S FORMULA FOR THE MATRICES OF GENERALIZED OCTONIONS (CAYLEY NUMBERS)

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## ABSTRACT

This article focuses on the matrices of generalized octonions, also called Cayley numbers, which include real octonions, split octonions, semi octonions, split semi octonions, quasi octonions, split quasi octonions and para octonions in special cases. We give the matrix representations of the generalized octonions with some algebraic properties. Also, we present formulae for powers and roots of the matrix representations.

**Keywords** Cayley numbers · 3-parameter generalized octonions · De Moivre formula

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