ICMASE 2026

A GENERALIZATION OF (AMPLY) COFINITELY G-SUPPLEMENTED MODULES

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Abstract

In this work, every ring has an identity and every module over a ring R is a unitary left R-module. Let M be an R-module and $U, V \leq M$. If $M = U + V$ and $M = U + T$ with $T \trianglelefteq V$ implies that $T = V$, or equivalently, $M = U + V$ and $U \cap V \ll_g V$, then V is called a g-supplement of U in M. M is said to be g-supplemented if every submodule of M has a g-supplement in M. M is said to be cofinitely g-supplemented if every cofinite submodule of M has a g-supplement in M. Let M be an R-module and $U \leq M$. If for every $V \leq M$ such that $M = U + V$, U has a g-supplement $V^{'}$ in M with $V^{'} \leq V$, then we say U has ample g-supplements in M. If every submodule of M has ample g-supplements in M, then M is called an amply g-supplemented module. If every cofinite submodule of M has ample g-supplements in M, then M is called an amply cofinitely g-supplemented module. Let M be an R-module and T be a maximal submodule of M. If $T \trianglelefteq M$, then T is called a g-maximal submodule of M. The intersection of all g-maximal submodules of an R-module M is called the generalized radical (briefly, g-radical) of M and denoted by $Rad_g M$. If M have no g-maximal submodules, then the g-radical of M is defined by $Rad_g M = M$. Let M be an R-module and $U, V \leq M$. If $M = U + V$ and $U \cap V \leq Rad_g V$, then V is called a g-radical supplement of U in M. If every submodule of M has a g-radical supplement in M, then M is called a g-radical supplemented module. M is said to be cofinitely g-radical supplemented if every cofinite submodule of M has a g-radical supplement in M. Let M be an R-module and $U \leq M$. If for every $V \leq M$ such that $M = U + V$, U has a g-radical supplement $V^{'}$ in M with $V^{'} \leq V$, then we say U has ample g-radical supplements in M. If every submodule of M has ample g-radical supplements in M, then M is called an amply g-radical supplemented module. If every cofinite submodule of M has ample g-radical supplements in M, then M is called an amply g-radical supplemented module. In this work, some new properties of (amply) cofinitely g-radical supplemented modules are investigated. It is clear that every (amply) cofinitely g-supplemented module is (amply) cofinitely g-radical supplemented.

Keywords Essential Submodules · Small Submodules · g-Radical · g-Supplemented Modules

2020 Mathematics Subject Classification: 13C13, 13C60.

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