
NEW GRÖNWALL-FREDHOLM TYPE INEQUALITY WITH APPLICATION TO A LARGE CLASS OF NONLINEAR FRACTIONAL DIFFERENTIAL EQUATIONS

Naoufel Hatime ^{1,*}, M'hamed Elomari ¹, Ali El Mfadel ^{1,2}, Said Melliani ¹

¹Laboratory of Applied Mathematics and Scientific Computing, Sultan Moulay Slimane University, PO Box 532, Beni Mellal, 23000, Morocco.

²Superior School of Technology, Sultan Moulay Slimane University, Khenifra, Morocco

ABSTRACT

In this work, we consider a Ψ -Caputo fractional Newton's Second Law of motion involving the ψ -Caputo operator of order $\alpha \in (1, 2]$. We prove the existence and uniqueness of solutions for different classes of force functions f acting on a specific object in motion. We introduced a new Grönwall-Fredholm type inequality ; a generalization of well-known Pachpatte's result, but under a weaker condition, that will help to prove Ulam-Hyers-Rassias stability \mathbb{UHR} and generalized Ulam-Hyers-Rassias \mathbb{GUHR} stability of the solutions for a large class of fractional differential equations, mainly the cases where the solutions related to a Green function. In addition, we replace the well-known used condition;

- μ is a continuous function over $[a, b]$,
- μ is an increasing function over $[a, b]$,
- There exists a positive constant γ_μ such that

$${}^f I_{a+}^\alpha \mu(t) \leq \gamma_\mu \mu(t), \forall t \in [a, b].$$

where ${}^f I_{a+}^\alpha$ denote a fractional integral.

by a new condition that is considerably simpler and weaker ; μ only a positive continuous function with $\inf_{s \in [a, b]} \mu(s) > 0$. Finally, we provide a example to establish our theoretical results.

Keywords Fractional differential equations · ψ -Caputo derivative · Grönwall type inequality · Ulam-Hyers stability

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* Corresponding Author's E-mail: hatime.naoufel@usms.ma