VI International Conference on Mathematics and its Applications in Science and Engineering (ICMASE 2025)

EIGENVECTORS AND EIGENVALUES OF THE EFFECTIVE RESISTANCE MATRIX OF A GRAPH

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ABSTRACT

We consider undirected, weighted and connected graphs on N nodes, whose corresponding graph-related matrices are symmetric. The graph G contains a set $\mathcal{N}$ of N nodes and a set $\mathcal{L}$ of L links. As mentioned in my book [2], I believe that, after the adjacency matrix A and Laplacian matrix Q of a graph G, the effective resistance matrix Ω with elements ω_{ij} is the third important matrix associated with graph G. The effective resistance matrix Ω is closely related to the Laplacian matrix by

$$\Omega = \zeta u^T + u\zeta^T - 2Q^\dagger \tag{1}$$

where u is the all-one vector, the vector $\zeta = \left(Q^\dagger_{11}, Q^\dagger_{22}, \ldots, Q^\dagger_{NN}\right)$ and $Q^\dagger$ is the pseudoinverse of the Laplacian [3], [2, Secion 4.2]. The effective resistance matrix Ω is a distance matrix [2, art. 8]. Here, we explicitly express the eigenvectors $v_1, v_2, \ldots, v_N$ and eigenvalues $\rho_1, \rho_2, \ldots, \rho_N$ of the effective resistance matrix Ω in terms of the eigenvectors $z_1, z_2, \ldots, z_N = \frac{u}{\sqrt{N}}$ and eigenvalues $\mu_1 \geq \mu_2 \geq \ldots \geq \mu_N = 0$ of the possibly weighted, but symmetric Laplacian Q. We also deduce the exact characteristic polynomial and thus improve on a famous interlacing result by Fiedler [1, Corollary 6.2.9], [2, Theorem 33].

Keywords Graphs · Spectrum · Electric currents

References

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