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On Various Alpha-type Convergences and Korovkin-type Theorems

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Abstract

This study considers strong versions of alpha convergence (continuous convergence), namely uniform alpha convergence and Cauchy alpha convergence, for sequences of functions defined between metric spaces. Since alpha convergence of function sequences guarantees the continuity of the limit function, these stronger convergences ensure the uniform continuity and Cauchy continuity of the limit function. Moreover, we examine their connections with existing notions with respect to the domain of the functions and the terms of the function sequences. Furthermore, Korovkin-type approximation theorems are established with respect to these new convergences.

Definition 1. [1] The sequence of functions (f_n) is called *alpha convergent* f on X if for all $x \in X$ and for every sequence $(x_n) \subset X$ with $x_\to x$, we have $f_n(x_n) \to f(x_0)$

Definition 2. [2] The sequence of functions (f_n) is called *uniform alpha convergent* f on X if for every sequences $(x_n), (y_n) \subset X$ with $d(x_n, y_n) \to 0$, we have $\rho\big(f_n(x_n), f(y_n)\big) \to 0$.

Definition 3. [2] The sequence of functions (f_n) is called *Cauchy alpha convergent* f on X if for every Cauchy sequences $(x_n), (y_n) \subset X$ with $d(x_n, y_n) \to 0$, we have $\rho\big(f_n(x_n), f(y_n)\big) \to 0$.

Keywords Function sequence · Alpha convergence · Korovkin-type theorems.

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