
NECESSARY OPTIMALITY CONDITIONS FOR DISCRETE SYSTEMS DESCRIBED BY GRADIENT-TYPE HAMILTONIAN

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ABSTRACT

The present study introduces a modern variational framework for processes characterized by discrete evolution. Within a class of perturbed functions governed by gradient-type variations, we investigate the necessary optimality criteria for discrete multidimensional optimal control problems involving higher-dimensional recurrence relations. First, we extend several concepts from the continuous-time setting to the discrete framework. Concretely, first-order Lagrangians, classical differential operators, complete controllability conditions and equality-type constraints are reconsidered through discrete formulations. Thereafter, we formulate the model under study by incorporating first-order higher-dimensional iterative processes together with discrete gradient-type variations. Our main results establish a discrete Pontryagin maximum principle and derive the associated Hamiltonian-based necessary optimality conditions. Notably, the introduction of gradient-type variations leads to partial differential equations whose right-hand side is governed by a real constant, yielding a more comprehensive and versatile description of the system's dynamics, as these equations incorporate structural information beyond what the classical formulation can capture.

Keywords Discrete optimal control · Multi-directional recurrence relations · Discrete gradient-type variations · Pontryagin maximum

References

- [1] Dragu, M. A., Treanță, S., Optimal control models with gradient-type variations constrained by third-order PDEs, *Communications in Nonlinear Science and Numerical Simulation*, 109947, 2026.
- [2] Treanță, S., Dragu, M. A., Euler–Lagrange equation for gradient-type Lagrangian and related conservation laws, *Journal of Applied Mathematics and Computing*, 71, 1565–1579, 2024.
- [3] Bejenaru, A., Pîrvan, M., Discrete Multivariate Optimal Control, *Journal of Optimization Theory and Applications*, 180(2), 442-450, 2019.

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