VII International Conference on Mathematics and its Applications in Science and Engineering
(ICMASE 2026)

ICMASE 2026

AN EFFECTIVE FINITE VOLUME METHOD FOR NONLINEAR MARSHAK WAVE PROBLEMS

Ece H. Korkmaz[1,*], Samet Y. Kadioglu[1], Ersin Ozugurlu[1]

[1] *Department of Mathematical Engineering, Istanbul Technical University, Istanbul, Turkey*

ABSTRACT

In high-energy-density physics, from inertial confinement fusion to astrophysical plasmas, heat is carried primarily by radiation rather than by material conduction. The photon absorption cross-section decreases sharply with temperature, allowing radiation to diffuse much more readily through hot material than through cold material [1]. Under the equilibrium-diffusion approximation, the coupled radiation-diffusion system reduces to a single nonlinear diffusion equation whose temperature-dependent conductivity degenerates as the medium cools. The resulting equation is degenerate parabolic, leading to finite-speed propagation and the formation of a sharp radiation front, in contrast to the instantaneous spreading characteristic of linear diffusion. The Marshak wave problem, in which a radiation source at the boundary drives such a front into a cold medium, provides a classical benchmark for this behavior [2, 3]. Accurately resolving this front is computationally challenging. The strong nonlinearity and degeneracy motivate the use of implicit schemes, which, although unconditionally stable, require solving a large nonlinear algebraic system at every time step. Consequently, the overall computational cost depends strongly on the efficiency of the nonlinear solution procedure [4, 5]. To address this challenge, we develop a conservative finite volume framework coupled with a Jacobian-Free Newton-Krylov (JFNK) method for solving the resulting nonlinear systems. In this context, accurate and computationally efficient numerical methods are essential for reliably resolving the underlying physics and for providing a foundation for more complex radiation-diffusion simulations. The proposed approach is evaluated through mesh and time-step refinement studies, with particular emphasis on solution accuracy, convergence behavior, and computational cost, aiming to establish an effective balance between numerical accuracy and computational efficiency.

***K*eywords** Marshak waves · Nonlinear radiation diffusion · Finite volume method · JFNK method

References

[1] Zeldovich Ya.B., Raizer Yu.P., Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena, Academic Press, New York, 1967.

[2] Marshak R.E., Effect of radiation on shock wave behavior, Phys. Fluids, 1(1): 24–29, 1958.

[3] Hammer J.H., Rosen M.D., A consistent approach to solving the radiation diffusion equation, Phys. Plasmas, 10(5): 1829–1845, 2003.

[4] Zheng S., Tang M., Zhang Q., Xiong T., High order conservative LDG-IMEX methods for the degenerate nonlinear non-equilibrium radiation diffusion problems, J. Comput. Phys., 503: 112838, 2024.

[5] Kadioglu S.Y., Ozugurlu E., A Jacobian-Free Newton-Krylov Method to Solve Tumor Growth Problems with Effective Preconditioning Strategies, Appl. Sci., 13(11): 6579, 2023.

[*] *Corresponding Author's E-mail: korkmazece@itu.edu.tr*