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SEMI-ANALYTICAL SOLUTION OF THE (1+1)-DIMENSIONAL KAUP SYSTEM USING THE HOMOTOPY ANALYSIS METHOD

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ABSTRACT

The $(1+1)$-dimensional Kaup system is an important nonlinear evolution model that describes the propagation of long waves in shallow water and other dispersive physical phenomena. Owing to the strong nonlinearity of the governing equations, obtaining accurate analytical solutions remains a challenging task. In this work, the Homotopy Analysis Method (HAM) is employed to construct rapidly convergent semi-analytical solutions of the Kaup system without introducing perturbation parameters or discretization techniques. Appropriate auxiliary linear operators and initial approximations are selected to formulate the zeroth- and higher-order deformation equations, from which recursive solution series are obtained. The convergence-control parameter is determined using the averaged squared residual error approach, ensuring the convergence and stability of the generated solution series. Numerical computations demonstrate that the residual errors decrease significantly with increasing approximation order, confirming the effectiveness of the proposed methodology. The fourth-order HAM solution exhibits excellent agreement with the available exact solution, and the corresponding relative errors remain extremely small over the computational domain. Graphical comparisons further verify that the obtained semi-analytical solutions accurately reproduce the nonlinear wave behaviour described by the Kaup system. The present study highlights the flexibility, robustness, and computational efficiency of HAM for solving nonlinear partial differential equations arising in fluid dynamics and wave propagation. The proposed framework can be extended to more complex nonlinear models involving higher-dimensional systems, variable coefficients, fractional operators, and coupled wave equations encountered in mathematical physics and engineering applications.

Keywords $(1+1)$-dimensional Kaup system · Homotopy Analysis Method · Semi-analytical solution · Nonlinear partial differential equations · Convergence analysis.

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