

# ORBIT EXTRAPOLATION

Denis Hautesserres<sup>1,\*</sup>

<sup>1</sup>Private researcher, France

## ABSTRACT

Orbit extrapolation is the calculation of the trajectory of the orbit at time  $t$  from knowledge of its initial condition at time  $t_0$ . This calculation can be either analytical, ie. be an algebraic formula, a priori result of formal mathematical integration, at time  $t$ , or numerical, ie. be the integration of an ordinary differential equation (ODE) step by step from  $t_0$  to  $t$ . For many reasons this calculation is not easy to do. Outside of the unperturbed two-body problem, formal integration does not exist. Perturbed problems require high-order numerical integrators; n-body problems, for an orbital trajectory and  $n > 2$ , also require these efficient numerical integrators. Over the centuries and knowledge of orbital perturbations, the precision of orbit calculation has increased. However, a major challenge remains, that of the duration of the extrapolation. The further the extrapolation is in time,  $t \gg t_0$ , the more the result deteriorates to the point of no longer being good. The 21st century has so far seen a real improvement in orbit extrapolation, even for high and very eccentric orbits. While there is no formal solution to the real orbital trajectory, analytical methods can nevertheless be implemented for so-called average problems or in an approximation framework. Here we summarize some of the most efficient modern analytical and numerical orbit extrapolation methods of the perturbed two-body problem. We will begin by recalling the basics of orbital mechanics, as well as those of the numerical integration of ordinary differential equations. The aim is indeed this review of methods which seems useful to any mechanic in charge of choosing the method of calculating his orbit. This review is also useful to researchers in orbital mechanics by making them aware of methods that are not their own, but also to mathematics teachers. The presentation, although short but synthetic, is the result of years of study and research in the field of space technology. It is a challenge to expose so many techniques in a very short time, but the summary tables will help us.

	PERTURBATIONS	ACCURACY	COMPUTATION TIME	ERROR	EXAMPLE	DEDICATED TO (non-exhaustive)
<b>GENERAL PERTURBATIONS</b> (Analytical integration)	limited number of perturbations	low	very fast computing (at the date)	non-cumulative	SGP4	mission analysis, tracking, catalog
<b>SEMI-ANALYTICAL</b> (Analytical-numerical integration)	all essential perturbations	high	fast computing (big step)	long term cumulative (roundoff)	DSST	long term propagation
<b>SPECIAL PERTURBATIONS</b> (Numerical integration)	all the perturbations	high	long (large number of calculations)	cumulative (truncation, roundoff)	DOPRI853 + forces model	orbit determination

Figure 1: Summary of the three main classes of orbit extrapolation.

**Keywords** Mechanisc · ODEs · Orbit · Extrapolation · Analytical · Semi-Analytical · Numerical

\*E-mail: [denis.hautesserres@gmail.com](mailto:denis.hautesserres@gmail.com)