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ON SOME CLASSES OF GENERALIZED QUASI-EINSTEIN SPACETIMES WITH APPLICATIONS IN GENERAL RELATIVITY

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ABSTRACT

In general relativity, a spacetime, denoted by M^4 is viewed as a Lorentzian manifold equipped with a Lorentzian metric g. This metric has a signature of $(-,+,+,+)$, indicating the mixture of positive and negative signs. It allows for the presence of a vector that is time-oriented and valid globally throughout the spacetime.

A Lorentzian manifold M^4 is called a perfect fluid spacetime [1], if the non-zero Ricci tensor R_{ij} satisfies

$$R_{ij} = \alpha g_{ij} + \beta A_i A_j$$

where α, β are scalar functions and A_i is the non-zero 1-form named as "the generator of the manifold" and $A_i A^i = -1$.

For a perfect fluid spacetime, the energy momentum tensor T_{ij} is given by [2]

$$T_{ij} = (p+\sigma)A_i A_j + p g_{ij}$$

where σ and p denote the energy density and the isotropic pressure, respectively and A_i is a non-vanishing vector.

The Einstein field equations without cosmological constant are presented by

$$R_{ij} - \frac{R}{2} g_{ij} = kT_{ij}$$

where R denotes the scalar curvature and k indicates the gravitational constant.

In modern cosmology, dark energy is considered as a candidate to accelerate the expansion of the universe and the scalar functions σ and p are considered by an equation of state (EoS), $p = p(\sigma, T_0)$ that regulates the quality of the ideal fluid by denoting T_0 as the absolute temperature. If we take T_0 as a constant, then the (EoS) is reduced to $p = p(\sigma)$. Then, this spacetime is called isentropic [3].

From (EoS), a perfect fluid spacetime is referred as stiff matter if $p = \sigma$, dark matter era if $p = -\sigma$, dust matter era if $p = 0$, the radiation era if $p = \frac{\sigma}{3}$, cosmic walls if $p = -\frac{2\sigma}{3}$ and strings if $p = -3\sigma$ ([4]-[6]). If $\frac{p}{\sigma} < -\frac{1}{3}$ then the universe represents accelerating phase, if $-1 < \frac{p}{\sigma} < 0$ then the universe represents quintessence phase.

A generalization of Einstein manifolds is the generalized quasi-Einstein manifold ([7],[8]). The Ricci tensor of a generalized quasi-Einstein manifold satisfies the following condition

$$R_{ij} = \alpha g_{ij} + \beta A_i A_j + \gamma(A_i B_j + A_j B_i)$$

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where γ is a scalar , A_i is unit a time-like vector and B_i is a unit space-like vector. Also, A_i and B_i are orthogonal vectors. A Lorentzian manifold whose Ricci tensor satisfies the last equation is called generalized quasi-Einstein spacetime.

In this paper, some special conditions in a generalized quasi-Einstein spacetime are considered. Under some special conditions, the physical properties of these spacetimes are examined. Then, it is shown that such a turns into a perfect fluid or static spacetime or a special type of product spacetime under some assumptions.

In the last part of this study, the applications of the considered spacetime in general relativity are discussed.

***K*eywords** Generalized quasi-Einstein spacetime · perfect fluid · static spacetime · general relativity · expansion scalar · vorticity.

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