
FINITELY-VALUED MODAL LOGICS THAT CANNOT DISTINGUISH SERIAL FRAMES

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ABSTRACT

Badia, Caicedo and Noguera [1] proved that every class of crisp Kripke frames definable by validity in a modal logic valued in a finite lattice algebra is definable in classical modal logic, and that the converse holds whenever the algebra interprets a Boolean algebra through a unary term. Whether this Boolean-interpretation hypothesis is necessary was left as a central open problem. This talk presents a broad negative answer for natural families of many-valued algebras. The main tool is a collapse pre-order: a pre-order compatible with all basic operations, possessing a least element, and placing no element above the top truth value. For any algebra carrying such a pre-order, validity of a modal formula on a frame reduces to its value under a single least valuation. On serial Kripke frames, this value is independent of the frame. Consequently, these logics cannot distinguish any two serial frames, although classical modal logic can separate standard serial-frame subclasses, including reflexive, transitive, and symmetric frames. The construction applies to all order-preserving finite lattice algebras, including negation-free reducts of finite Łukasiewicz chains, and to non-Boolean De Morgan chains. The Belnap–Dunn four-valued algebra does not admit a collapse pre-order, but a twist-structure argument yields the same blindness to serial frames. For this algebra, and for every nontrivial monotone bound-preserving algebra with the bounds in its signature, the definable frame classes are exactly those determined by variable-free classical modal formulas. I also explain how an incorrect identity used in the literature for finite MV-algebras can be repaired without changing the earlier conclusions. These results show that some mechanism for recovering two-valued reasoning is indispensable for full classical frame definability, while leaving open whether Boolean interpretability gives the exact boundary.

Keywords Many-valued modal logic · Frame definability · Finite lattice algebra · Kripke frames · Algebraic logic

References

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