

---

# AUGMENTED MESH REFINEMENT IN PHYSICS-INFORMED NEURAL NETWORKS FOR EFFICIENT SOLUTION OF STIFF PARTIAL DIFFERENTIAL EQUATIONS

---

Nursyiva Irsalinda<sup>1,2</sup>, Maharani Abu Bakar<sup>1,3</sup> \*

<sup>1</sup>Faculty of Computer Science and Mathematics, Universiti Malaysia Terengganu, Kuala Nerus, Terengganu, 21030, Malaysia

<sup>2</sup>Faculty of Applied Sciences and Technology, Universitas Ahmad Dahlan, Banguntapan, Bantul, 55166, Indonesia

<sup>3</sup>Special Interest Group Modelling and Data Analytics, Faculty of Computer Science and Mathematics, Universiti Malaysia Terengganu, Kuala Nerus, Terengganu, 21030, Malaysia

## ABSTRACT

Physics-Informed Neural Networks (PINNs) have emerged as a promising approach for solving partial differential equations; however, their performance often degrades when applied to stiff PDEs due to the inability of uniform training meshes to accurately capture localized sharp gradients without incurring high computational cost. This limitation leads to inefficient learning and slow convergence, particularly in regions with highly nonlinear behavior. To address this issue, this study proposes an Augmented Mesh Refinement Physics-Informed Neural Network (Aug-MR PINN), a novel framework that dynamically reallocates training points toward regions with high residual errors. The proposed method employs residual-based criteria to adaptively refine the mesh during training, enabling the model to focus on critical regions while avoiding unnecessary computation in smoother areas. This adaptive strategy enhances the representation of localized features and improves overall training efficiency. Experimental results demonstrate that Aug-MR PINN consistently achieves lower loss values and faster convergence compared to existing approaches, including AMR-FEM, standard PINN, R-PINN, and X-PINN. Furthermore, the proposed refinement strategies exhibit comparable performance, highlighting the robustness and flexibility of the framework. Overall, this work provides an effective and scalable solution for solving stiff PDEs and offers strong potential for broader applications in computational physics, including fluid dynamics, wave propagation, and multiphase systems.

**Keywords** Mesh refinement · Physics Informed Neural Networks · Stiff PDE · Mesh grid

## References

- [1] Raissi M., Perdikaris P., Karniadakis G. E., Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations, *J. Comput. Phys.*, 378: 686-707, 2019.
- [2] Pratama D. A., Bakar M. A., Ibrahim N. F., Idris R., Mohamed N., Physical restriction neural networks with restarting strategy for solving mathematical model of thermal heat equation for early diagnose breast cancer, *Results Appl. Math.*, 19: 100384, 2023.
- [3] Jagtap A. D., Karniadakis G. E., Extended physics-informed neural networks (XPINNs): A generalized space-time domain decomposition based deep learning framework for nonlinear partial differential equations, *Commun. Comput. Phys.*, 28(5): 2002-2041, 2020.
- [4] Irsalinda N., Bakar M. A., Harun F. N., Surono S., Pratama D. A., A new hybrid approach for solving partial differential equations: Combining physics-informed neural networks with extreme learning machines, *Results Appl. Math.*, 21: 100443, 2024.

---

\*Corresponding Author's E-mail: maharani@umt.edu.my

- [5] De Florio M., Schiassi E., Furfaro R., Physics-informed neural networks and functional interpolation for stiff chemical kinetics, *Chaos*, 32(6): 063126, 2022.