NEW APPROACH FOR SOLVING THE MATRIX DIFFERENTIAL EQUATION OF THE SECOND ORDER. APPLICATIONS

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ABSTRACT

Linear matrix differential equations of the second order play a crucial role in various fields of pure and applied mathematics, such as the calculus, the control theory and the engineering, as well as in other fields of exact and applied sciences (see, for instance, [2, 4, 6, 7], and references therein). Apostol (see [1]) and Kolodner (see [3]) have studied the following matrix differential equation of the second-order

$$X^{''}(t) = AX(t), \tag{1}$$

submitted to the initial data $X(0)$ and $X'(0)$, where $A \in \mathbb{C}^{d\times d}$ the algebra of square matrices of order $d \times d$. Apostol and Kolodner gave some properties of the solutions of Equation (1). In [1], Apostol proposes a process for solving equation (1) based on Putzer's method to calculate the exponential of matrices (see [1, 5]). Furthermore, the second order linear matrix differential equations appear in several mechanical topics, among them the mass-spring differential systems.

This talk concerns the generalized Apostol-Kolodner differential equations of the second order. Our approach is based on some matrix square root properties, the Fibonacci-Hörner decomposition of matrix powers, and its related dynamical solution. Various explicit compact formulas for the solutions of the generalized Apostol-Kolodner matrix differential equations are established. Finally, to validate the results and show their robustness, examples and application the mass-spring differential systems are provided.

Keywords Linear matrix differential equations · Apostol-Kolodner equation · Fibonacci-Hörner decomposition · Dynamical solution · Companion block matrix · Mass-spring differential systems.

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