ON GEOMETRY OF ASSOCIATED CURVES OF TIMELIKE MAGNETIC CURVES

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ABSTRACT

The study of magnetic curves and vector fields represents a significant area of research in both the fields of physics and mathematics. A fundamental property of any magnetic vector field is that it is a divergence-free vector field in three spaces. Magnetic vector fields are generated by electric currents and changing electric fields, and charged particles moving within these fields are influenced by magnetic forces. The paths traced by these particles, known as magnetic trajectories, are referred to as magnetic curves. The study of magnetic curves is crucial for both theoretical physics and engineering applications, including the analysis of electromagnetism, plasma physics, and magnetic materials. Also, the issue of investigating magnetic trajectories initially appears to be a purely physical problem, recent studies have demonstrated that the characterization of magnetic flow in a magnetic field has introduced a variational perspective that is more geometrical in nature. In particular, magnetic curves have been developed through the application of techniques derived from differential geometry and methods of the calculus of variation, from basic spaces to manifolds. The tangent, normal, and binormal vectors characterize the kinematic and geometric aspects of a particle's motion, influencing the path of a charged particle in a magnetic field. Additionally, the time dimension plays a role in shaping its trajectory. In this paper we first investigate whether there are Mannheim, Bertrand and involute-evolute curve pairs for a timelike N-magnetic curve in 3-dimensional Minkowski space. The relationships between this curve and the curve pairs are obtained. In this way some geometric properties between the curves are derived. Similar procedures are carried out for timelike B-magnetic curves and important results are obtained.

***K*eywords** Killing vector fields · Lorentz force equation · Magnetic trajectory.

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