ICMASE 2026

Nonlinear Coupled Fractional Boundary Conditions with Variable-Order Caputo–Fabrizio and p-Laplacian Operators

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Abstract

In this paper, we investigate a coupled system of boundary value problems for variable-order Caputo–Fabrizio fractional differential equations involving the nonlinear p-Laplacian operator:

$$\begin{cases} {}_0^{CF}D_t^{\alpha_1(t)}\big(\phi_p\, {}_0^{CF}D_t^{\beta_1(t)}\mathcal{U}(t)\big) - \mathcal{G}_1(t)\mathcal{U}(t) = \psi_1\big(t, \mathcal{U}(t), {}_0^{C}D_t^{\gamma_1}\mathcal{U}(t)\big), \\ {}_0^{CF}D_t^{\alpha_2(t)}\big(\phi_p\, {}_0^{CF}D_t^{\beta_2(t)}\mathcal{V}(t)\big) - \mathcal{G}_2(t)\mathcal{V}(t) = \psi_2\big(t, \mathcal{V}(t), {}_0^{C}D_t^{\gamma_2}\mathcal{V}(t)\big), \end{cases}$$

subject to the boundary conditions

$$\mathcal{U}(0) = 0 \quad {}_0^{C}D_t^{\gamma_1}\mathcal{U}(0) = {}_0^{C}D_t^{\gamma_1}\mathcal{U}(1) \qquad \mathcal{V}(0) = 0 \quad {}_0^{C}D_t^{\gamma_2}\mathcal{V}(0) = {}_0^{C}D_t^{\gamma_2}\mathcal{V}(1),$$

where ${}_0^{CF}D_t^{\eta(t)}$ denotes the Caputo–Fabrizio derivative of variable order $\eta(t) \in (0,1)$, while ${}_0^{C}D_t^{\gamma_i}$ stands for the Caputo derivative of order $\gamma_i \in (0,1]$. We assume $1 < \alpha_i(t) + \beta_i(t) \leq 2$ for $i = 1, 2$. The operator $\phi_p(r) = |r|^{p-2}r$ represents the generalized p-Laplacian with $p > 1$. The functions $\mathcal{G}_i : [0,1] \to \mathbb{R}$ are continuous and the nonlinearities ψ_i satisfy Lipschitz-type conditions with respect to their state variables.

Using fixed-point theorems and Green's function techniques, we establish the existence and uniqueness of solutions. Moreover, we study the Hyers–Ulam stability under the same assumptions. These results extend recent contributions on fractional boundary value problems for nonlinear coupled systems.

***K*eywords** Fractional derivatives · p-Laplacian operator · Variable-order Caputo–Fabrizio derivative · Boundary value problems · Hyers–Ulam Stability

Acknowledgment

This research was funded in whole by the National Science Center, Poland, grant number 2023/51/B/ST8/01062. For the purpose of Open Access, the authors have applied a CC-BY public copyright license to any Author Accepted Manuscript (AAM) version arising from this submission.

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