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A GENERALIZATION OF BETA-STAR RELATION IN MODULES

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ABSTRACT

In this work, every ring has an identity and every module over a ring R is a unitary left R-module. Let M be an R-module. It is defined the relation $'\beta^{*\prime}$ on the set of submodules of an R-module M by $X\beta^{*}Y$ if and only if $Y+K=M$ for every $K\leq M$ such that $X+K=M$ and $X+T=M$ for every $T\leq M$ such that $Y+T=M$. Let M be an $R-$module and $K\leq V\leq M$. We say V lies above K in M if $V/K\ll M/K$. Let M be an R-module. It is defined the relation $'\beta_{g}^{*\prime}$ on the set of submodules of an R-module M by $X\beta_{g}^{*}Y$ if and only if $Y+K=M$ for every $K\trianglelefteq M$ such that $X+K=M$ and $X+T=M$ for every $T\trianglelefteq M$ such that $Y+T=M$. Let M be an R-module and $F\leq M$. A submodule K of M is called an F-small (or F-superfluous) submodule of M and written by $K\ll_{F}M$ if $T=M$ for every $T\leq M$ such that $F\leq T$ and $M=K+T$. A submodule T of M is said to be F-maximal if T is a maximal submodule of M and $F\leq T$. The intersection of all F-maximal submodules of M is called the F-radical of M and denoted by $Rad_{F}M$. If M have no F-maximal submodules, then the F-radical of M is defined by $Rad_{F}M=M$. Let M be an R-module and $F,U,V\leq M$. V is called an F-supplement of U in M if V is minimal with respect to $F\leq L\leq M$ and $M=U+L$. V is an F-supplement of U in M if and only if $M=U+V$ and $U\cap V\ll_{F}V$. If every submodule of M has an F-supplement in M, then M is called an F-suplemented module. Let M be an R-module and $F,U\leq M$. If for every $V\leq M$ such that $M=U+V$, U has a F-supplement $V^{'}$ with $V^{'}\leq V$, we say U has ample F-supplements in M. If every submodule of M has ample F-supplements in M, then M is called an amply F-supplemented module. Let M be an $R-$module and $F\leq M$. It is defined the relation $'\beta_{F}^{*\prime}$ on the set of submodules of the module M by $X\beta_{F}^{*}Y$ if and only if $Y+K=M$ for every $K\leq M$ such that $F\leq K$ and $X+K=M$ and $X+T=M$ for every $T\leq M$ such that $F\leq T$ and $Y+T=M$. In this work, some new properties of $'\beta_{F}^{*\prime}$ relation are investigated. $'\beta_{F}^{*\prime}$ relation is a generalization of $'\beta^{*\prime}$ relation and lying above in modules.

Keywords Modules · β^{*} relation · F-Small Submodules · F-Supplemented Modules

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