THE INVISIBLE STRUCTURE OF DATA: A GEOMETRIC-ALGEBRAIC READING OF MODE

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ABSTRACT

The connections between the several disciplines of Mathematics - e.g., Algebra, Geometry, Calculus, Statistics, and Arithmetic - are essential because they transform sometimes fragmented knowledge into structured, applicable knowledge, overcoming the view that mathematics is merely a set of more or less isolated contents. This is valued in various mathematics curricula across different educational levels. The connections among mathematical knowledge across numerous areas deepen understanding, stimulate logical reasoning, and promote the practical application of mathematics to solve complex real-world problems. These connections allow students to apply, e.g., numerical relationships to shapes, link patterns to algebra, and use data analysis to interpret social or scientific situations. On the other hand, visualization is a vital, proven-effective tool in mathematics education. Visualization transforms abstract concepts into concrete, understandable, and assimilable images. So, mathematics visualization promotes a deeper conceptual understanding, improves problem-solving skills, and increases student engagement, allowing them to see patterns, make connections, and develop mathematical intuition. Additionally, it has a significant positive impact on mathematics learning outcomes. By using diagramming, manipulatives, and digital tools, teachers can make mathematics an active and creative experience, thereby improving students' performance and promoting a deeper, more intuitive understanding of the subject. Within this framework, this work aims to connect Statistics, Geometry, and Algebra and to promote the visualization, through technological tools, of the knowledge involved in this connection among these mathematical areas. In particular, the objective is to establish a relationship between the statistical concept of mode (continuous data case) and the geometric figure of the parabola, mediated by algebra, including the possibility of solving systems using matrices, and complemented by the use of technology - dynamic geometry, both for visualizing the aforementioned relationship and some processes for obtaining associated formulas. In this way, the statistical concept of the mode (in the continuous case) and two distinct geometric methods for estimating this parameter, based on proportionality relationships between similar triangles, are presented, using dynamic geometry to illustrate the model-building process and its differences. Based on one of the processes for estimating the mode in those data grouped by classes, this presentation demonstrates that the abscissa of the parabola's vertex equals the mode. This equality is verified geometrically using dynamic geometry and algebraically using systems of equations. The process and the relationship are visualized, formulas and their origins are confirmed, and a range of mathematical knowledge from several levels and areas of mathematics is connected. This presentation can serve as a starting point for teachers to engage students in a deeper understanding of intuitive, interconnected mathematics. Of course, it will also stimulate logical reasoning and awareness of the practical applicability of mathematics in solving more complex and/or abstract problems, and, through a visual basis, to make mathematics an active and creative experience, aiming to improve student performance.

Keywords Statistics · Geometry · Algebra · Mode · Algebraic linear systems · Mathematical connections · Visualization · Dynamic geometry

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