
GENERALIZED M-SPECTRAL PROBLEM HAVING HYDROGEN ATOM POTENTIAL

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ABSTRACT

In this paper, us address generalized \mathcal{M} -Sturm-Liouville problem with hydrogen atom potential associated with initial conditions:

$$-D_{\mathcal{M}}^{\alpha,\beta,\gamma} D_{\mathcal{M}}^{\alpha,\beta,\gamma} y(x) + \left[\frac{l(l+1)}{x^2} - \frac{2}{x} + q(x) \right] y(x) = \lambda y(x). \quad (1)$$

In this study, our aim we obtain representation of solution for generalized \mathcal{M} -Sturm-Liouville problem with hydrogen atom potential by taking advantage of Laplace transform by means of generalized \mathcal{M} -derivative.

Additionally, us define Laplace transform by means of generalized \mathcal{M} -derivative as follows:

$$\mathcal{L}_{\alpha,\beta,\gamma}\{f(t)\}(s) = \left[\frac{c_1 \dots c_r \Gamma(\beta + \gamma)}{a_1 \dots a_p \Gamma(\gamma)} \right] \int_0^\infty e^{-s} \frac{\left[\frac{c_1 \dots c_r \Gamma(\beta + \gamma)}{a_1 \dots a_p \Gamma(\gamma)} \right] t^\alpha}{\alpha} f(t) t^{\alpha-1} dt. \quad (2)$$

This problem, important results are demonstrated thanks to graphs of use MATLAB program [1-6].

Keywords \mathcal{M} -series · Laplace transform · Hydrogen atom potential · Generalized \mathcal{M} -derivative

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