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VII International Conference on

MATHEMATICS AND ITS APPLICATIONS IN SCIENCE AND ENGINEERING

ABSTRACT BOOK

14–17 September 2026

Nagoya University
Nagoya • Japan

Hybrid Conference

ABSTRACT BOOK

VII International Conference on Mathematics and Its Applications in Science and Engineering (ICMASE 2026)

Nagoya University

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Preface

This abstract booklet includes the abstracts of the papers that have been presented at VII International Conference on Mathematics and its Applications in Science and Engineering (ICMASE 2026) which was held in Nagoya University, Japan between 14-17 September, 2026, via hybrid. The aim of this conference is to exchange ideas, discuss developments in mathematics, develop collaborations and interact with professionals and researchers from all over the world about some of the following interesting topics: Functional Analysis, Approximation Theory, Real Analysis, Complex Analysis, Harmonic and non-Harmonic Analysis, Applied Analysis, Numerical Analysis, Geometry, Topology and Algebra, Modern Methods in Summability and Approximation, Operator Theory, Fixed Point Theory and Applications, Sequence Spaces and Matrix Transformation, Spectral Theory and Differential

Operators, Boundary Value Problems, Ordinary and Partial Differential Equations, Discontinuous Differential Equations, Convex Analysis and its Applications, Optimization and its Application, Mathematics Education, Applications on Variable Exponent Lebesgue Spaces, Applications on Differential Equations and Partial Differential Equations, Fourier Analysis, Wavelet and Harmonic Analysis Methods in Function Spaces, Applications on Computer Engineering, and Flow Dynamics. However, the talks are not restricted to these subjects.

Thanks to all committee members.

We wish everyone a fruitful conference and pleasant memories from ICMASE 2026.

Prof. Dr. Tomohiro SOGABE

Prof. Dr. Fatih YILMAZ

Chairs, ICMASE 2026

VII International Conference on Mathematics and Its Applications in Science and Engineering (ICMASE 2026)

Nagoya University,
14-17 September 2026

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- Luis HERNÁNDEZ ÁLVAREZ , Spanish National Research Council, (Spain)

  Title: AI attacks on post-quantum cryptographic systems

- Kota TAKEDA , Nagoya University, (Japan)

  Title: Ensemble Data Assimilation in High-Dimensional Chaotic Systems: Exploiting Low-Dimensional Structures

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VII International Conference on Mathematics and Its Applications in Science and Engineering

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Title: Inverse Eigenvalue Problems for Structured Matrices and Discrete Integrable Systems

Kanae Akaiwa

Kyoto Sangyo University, Japan

Abstract: An inverse eigenvalue problem (IEP) is an important research topic in the fields of science and engineering. In an IEP, how to construct matrices with prescribed eigenvalues and specific structures are discussed. Integrable systems are a class of nonlinear dynamical systems, and their time discretizations are called discrete integrable systems. In particular, (discrete) integrable systems possess conserved quantities and exact solutions. In this talk, we present methods for solving IEP for certain structured matrices by utilizing such “nice” properties of discrete integrable systems.

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Title: Ensemble Data Assimilation in High-Dimensional Chaotic Systems: Exploiting Low-Dimensional Structures

Kota Takeda

Nagoya University, Japan

Abstract: Data assimilation is a framework that combines mathematical models with observational data, originally developed in numerical weather prediction. We present its mathematical foundations and then focus on ensemble-based algorithms that exploit the essential low-dimensional structures of high-dimensional chaotic systems arising in atmospheric models. Recent advances are highlighted, and future directions are discussed.

Title: AI Attacks on Post-Quantum Cryptographic Systems

Luis Hernández Álvarez

Spanish National Research Council, Spain

Abstract: The National Institute of Standards and Technology has recently published the new Post–Quantum standards for Key Encapsulation Mechanism – ML-KEM - and Digital Signature – ML-DSA, FN-DSA, and SLH-DSA-. Following the standardization process, it is important to analyze the weaknesses and potential information leakages of these new standards, so that they can be prevented. This talk is focused on the importance of side channel attacks based on artificial intelligence models against the original proposals on which the standards are based - CRYSTALS-Kyber, CRYSTALS Dilithium, Falcon, and Sphincs+-. The principal artificial intelligence algorithms used in these attacks will be discussed, as well as common configurations, parameter selection, and other relevant tools that might be useful in this application.

CONTRIBUTED TALKS

ON LEONARDO NUMBERS AND THEIR MATRIX REPRESENTATION

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ABSTRACT

In this paper, we introduce and investigate a Hessenberg matrix T_n, which incorporates both local interactions of tridiagonal matrices and fixed-distance non-local couplings, thereby exhibiting a hybrid Hessenberg–tridiagonal structure. We establish a recurrence relation for its determinant and demonstrate that $\det(T_n)$ coincides with the Leonardo numbers, with further connections to Fibonacci numbers and Chebyshev polynomials. The characteristic polynomial of T_n is derived through a Schur complement approach, and its spectral properties are examined in detail. The eigenvalue distribution reveals non-Hermitian features, including the presence of complex conjugate pairs and oscillatory instabilities. Moreover, an explicit LU factorization is obtained, enabling efficient computation of determinants and shedding light on the sparse structural properties of the matrix. Numerical experiments and visualization of eigenvalue behavior confirm the theoretical findings and highlight the potential of T_n as a model for structured non-Hermitian operators relevant in dynamical systems, wave propagation, and transport phenomena.

***K*eywords** Hessenberg matrix · Determinant formulas · Leonardo numbers · Chebyshev polynomials · Eigenvalue analysis · LU decomposition

Acknowledgement The second author has been supported by the Scientific and Technological Research Council of Türkiye (TUBITAK)/2224-A - Grant Program for Participation in Scientific Meetings.

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TYPE 1 OCTONION NUMBERS AND SOME SYMBOLIC COMPUTATIONS

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ABSTRACT

In this presentation, we will define Type 1 Octonion Numbers. We will examine the algebraic structure and geometric properties of the set of Type 1 Octonion Numbers. We will derive De Moivre formulas. In addition, we will develop symbolic computations in MATLAB to support and illustrate the theoretical results.

***K*eywords** Type 1 Octonion Numbers · De Moivre Formula · Symbolic Computation

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ACCELERATING THE SOLUTION OF MULTIPLE SHIFTED SKEW-SYMMETRIC SYSTEMS VIA RANDOMIZED DEFLATION PRECONDITIONING

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ABSTRACT

We consider the efficient solution of multiple shifted skew-symmetric linear systems

$$(\alpha_i I + S)x_i = b_i, \quad i = 1, \ldots, m,$$

where all systems share the same skew-symmetric matrix S but differ in the shift parameters α_i. Such systems arise in a variety of scientific and engineering applications. When some shift parameters are small, the corresponding systems become highly ill-conditioned, making iterative solvers expensive. Solving each shifted system independently often leads to substantial computational redundancy, since the systems share the same skew-symmetric matrix S and exhibit closely related spectral structures. To exploit this property, we propose a randomized deflation preconditioning framework for multiple shifted skew-symmetric systems. The proposed method constructs a low-dimensional approximation of the dominant invariant subspace only once by applying a randomized Nyström approximation. This spectral information is then reused to efficiently build deflation preconditioners for all shifted systems with negligible additional cost. We derive theoretical bounds for the effective condition numbers of the preconditioned systems. Numerical experiments on large-scale problems demonstrate that the proposed approach significantly accelerates GMRES convergence and substantially reduces the overall setup cost compared with constructing separate preconditioners for individual shifts.

Keywords multiple shifted skew-symmetric linear systems · randomized deflation preconditioning · Nyström approximation · GMRES

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A Mathematical Model for Sustainable Multi-Item Inventory Systems with Demand Dynamics, Deterioration, and Carbon Emissions

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Abstract

Inventory management plays a critical role in modern supply chains, particularly for products that are subject to deterioration and whose demand changes over time. In addition, growing environmental concerns have increased the need for sustainable inventory policies that not only minimize operational costs but also reduce environmental impacts. This research focuses on developing mathematical models for sustainable multi-item inventory systems that integrate demand dynamics and carbon emissions into inventory decision-making. The proposed models consider inventory-dependent and time-dependent demand patterns, which are commonly observed in perishable products such as fruits, vegetables, dairy products and meat. Inventory-dependent demand reflects the influence of stock availability on customer purchasing behavior, while time-dependent demand captures changes in demand over the product lifecycle. Furthermore, the models incorporate carbon emissions generated from storage and transportation activities, enabling a comprehensive evaluation of both economic and environmental performance. We consider several replenishment policies, including individual ordering, joint replenishment, and hybrid approaches that combine elements of both. The objective is to determine the optimal replenishment interval and the inventory level at which inventory drops to zero that minimizes total inventory cost while accounting for environmental considerations. Analytical techniques are used to derive optimality conditions and investigate the structural properties of the models. We also develop an optimization algorithm to obtain optimal solutions. Sensitivity analysis is also conducted to assess the impact of changes in parameters on the model's optimal replenishment time, the time at which inventory drops to zero, and the total inventory cost. The expected outcomes of this research include new mathematical formulations for sustainable multi-item inventory systems, the structure of optimal inventory policies, and practical solution algorithms. By integrating demand dynamics, deterioration, and carbon emissions, this research advances green inventory systems that balance economic efficiency with environmental sustainability.

***K*eywords** Multi-Item Inventory Systems · Deteriorating Items · Demand Dynamics · Carbon Emissions · Sustainable Inventory Optimization

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PARAMETER ESTIMATION OF FRACTIONAL-ORDER DYNAMIC SYSTEMS USING PSO AND MACHINE LEARNING ALGORITHMS ON EMBEDDED DEVICES

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ABSTRACT

Fractional-order models offer a more accurate mathematical representation of complex industrial processes compared to classical integer-order models, owing to their ability to capture memory effects and hereditary dynamics through non-integer differential operators. However, their real-time identification remains a significant challenge, particularly in resource-constrained edge computing environments. This work presents a hybrid framework that combines Particle Swarm Optimization (PSO) with machine learning techniques (including neural networks, Support Vector Regression (SVR), and Extreme Gradient Boosting (XGBoost)) to estimate reduced-order Fractional First-Order Plus Dead-Time (FFOPDT) models from step response data. PSO is employed to generate high-fidelity parameter estimates by minimizing the discrepancy between the high-order system response and the reduced model output, while the trained AI-based algorithms enable deterministic and rapid inference suitable for real-time deployment. The proposed methodology is validated on a custom-built thermal system, demonstrating that neural networks achieve R^2 values above 0.99 with inference times below 10 milliseconds on NVIDIA Jetson edge devices, compared to median PSO execution times exceeding 300 seconds. These results highlight a clear trade-off between estimation accuracy and computational efficiency, establishing the hybrid framework as a robust solution for real-time fractional-order system identification in industrial edge computing applications.

***K*eywords** Fractional-order systems · Edge computing · Reduced-order modeling 3

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HIGHER ORDER NICKEL FIBONACCI NUMBERS AND SOME SYMBOLIC COMPUTATIONS

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ABSTRACT

In this presentation, we first define higher-order Nickel Fibonacci numbers. We then examine the algebraic properties of higher-order Nickel Fibonacci numbers. Finally, we construct higher-order Nickel Fibonacci matrices and study their algebraic properties. In addition, we will develop symbolic computations in MATLAB to support and illustrate the theoretical results.

***K*eywords** Higher Order Nickel Fibonacci Numbers · Higher Order Nickel Fibonacci Matrices · Symbolic Computations

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NULL SPATIAL SPLIT OCTONIONIC CURVES

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ABSTRACT

Split octonionic and spatial split octonionic valued functions of a single variable define curves in $\mathbb{R}_4^8$, the eight-dimensional Minkowski space of index 4. In [1], the authors used spatial split octonions to construct a new frame, called the G_2-frame, for non-null curves in $\mathbb{R}_4^7$. A key advantage of this frame is its computational simplicity and time efficiency compared to the classical Frenet frame. In this presentation, we examine the differential geometry of null curves in $\mathbb{R}_4^7$ using spatial split octonions. We construct a novel frame along these curves and derive the corresponding derivative formulas. Furthermore, we give symbolic computations for null spatial split octonionic curves with respect to this new frame.

***K*eywords** Split Octonions · Null Curves · Symbolic Computations

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GEODESIC CURVATURE RELATIONS OF CURVE PAIRS SHARING NORMAL DIRECTIONS ON S^2

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ABSTRACT

Focusing on the viewpoint of geodesic curvature, this paper explores the intrinsic geometric links between two curves rolling without slipping on the unit sphere S^2. The phenomenon of rolling motion without slipping introduces rigid structural constraints, which we systematically investigate by examining specific curve pairs characterized by a shared geodesic normal direction at their points of contact. Rather than employing standard kinematic formulations, this study adopts a purely differential geometric approach. First, the explicit coordinate-free relations and transformations between the geodesic Frenet frames of these matching curves are established. Building upon these fundamental frame transformations, we subsequently analyze and derive the exact mathematical relationships and identities governing their respective geodesic curvatures. Furthermore, we evaluate the geometric implications of these curvature dependencies under various rolling conditions. The theoretical framework developed herein not only provides deeper insights into the intrinsic geometry of spherical curves but also offers a robust mathematical alternative for analyzing constrained rotational motions on non-Euclidean surfaces.

***K*eywords** Spherical curves, Geodesic curvature, Motion

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SOME VARIOUS ASPECTS OF ANTISYMMETRIC QUASI-PSEUDO-METRICS

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ABSTRACT

The notion of antisymmetric quasi-pseudo-metric function which is in some sense opposite to the metric function appeared in the previous studies rather naturally, and the various aspects of antisymmetric functions had been investigated.

Accordingly, it would be natural to ask whether the family of antisymmetric quasi-pseudo-metrics may be complete and dense with respect to the symmetrization metric of a corresponding T_0 –quasi-metric not antisymmetric, described on the family of all quasi-pseudo-metrics. Hence, several asymmetric and topological properties of the family of antisymmetric functions, such as these, will be considered in this talk.

Keywords quasi-pseudo-metric · symmetrization topology · antisymmetric path · metric

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STRUCTURE-PRESERVING NUMERICAL SCHEMES FOR THE ALLEN–CAHN AND CAHN–HILLIARD MODELS WITH DYNAMIC BOUNDARY CONDITIONS

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ABSTRACT

The Allen–Cahn and Cahn–Hilliard equations are phase-field models that describe phase separation dynamics and can be derived as gradient flows of Ginzburg–Landau-type energies. To account for dynamics on the surface, dynamic boundary conditions such as Allen–Cahn-type [1] and Cahn–Hilliard-type [2] conditions have been introduced, extending these models to coupled bulk-surface systems.

In this work, we propose structure-preserving numerical schemes for the Allen–Cahn and Cahn–Hilliard models with dynamic boundary conditions. The proposed schemes preserve two important physical properties of the continuous systems: dissipation of the bulk-surface total energy and, whenever applicable, conservation of mass. We first reformulate the models within a variational framework and apply the finite element method for spatial discretization. For the temporal discretization, we follow the discrete gradient approach [3] and construct schemes that inherit the underlying energy structure of the continuous models at the discrete level.

The resulting schemes satisfy discrete energy dissipation laws independently of the choice of time-step size and preserve the corresponding mass conservation properties. The preservation of these physical properties is established theoretically and further confirmed through numerical experiments.

***K*eywords** Dynamic boundary conditions · Allen–Cahn equation · Cahn–Hilliard equation · Phase-field models · Structure-preserving numerical scheme · Energy stable scheme · Discrete gradient method

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PERFORMANCE EVALUATION OF ROBUST PRINCIPAL COMPONENT ANALYSIS-BASED MACHINE LEARNING MODELS FOR BREAST CANCER CLASSIFICATION

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ABSTRACT

Breast cancer classification remains an important problem in medical data analysis due to its high dimensionality and the presence of outliers that may affect model performance. Dimensionality reduction techniques such as Principal Component Analysis (PCA) are commonly used in high-dimensional data to reduce dimensionality while still retaining as much information as possible from the original data. Therefore, in this study, a performance evaluation of machine learning models incorporating PCA and Robust PCA for breast cancer classification is performed. Baseline models classifiers consisting of K-means, Support Vector Machine (SVM), and Artificial Neural Network (ANN) are compared with their corresponding PCA-based and Robust PCA-based classifiers. The objective of this study is to assess the impact of dimensionality reduction techniques on classification performance and robustness. All models are evaluated using accuracy, precision, recall, and F1-score. The experimental results show that SVM and Robust PCA-SVM have similar performance and are the best classifiers compared to other classifiers. In particular, Robust PCA-based machine learning models consistently demonstrate improved classification performance when compared to PCA-based approaches. These findings suggest that incorporating robust dimensionality reduction techniques enhances the reliability of classification outcomes in breast cancer data analysis. The study provides a comparison of baseline, PCA-based, and Robust PCA-based classifiers and highlights the benefits of robust dimensionality reduction in medical classification tasks.

***K*eywords** Principal Component Analysis · Robust Principal Component Analysis · Machine Learning · K-means · Support Vector Machine · Artificial Neural Network.

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UNLOCKING MULTICENTER INTEGRALS: A SIMPLIFIED CARTESIAN APPROACH TO SPHERICAL HARMONICS

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ABSTRACT

This study introduces a systematic derivation of Cartesian expressions for both surface and regular solid spherical harmonics, leveraging the efficiency of binomial coefficients. While spherical harmonics are fundamental to representing atomic and molecular symmetries, their traditional forms often pose significant computational challenges in multi-center contexts. Our approach transforms these complex functions into a more manageable algebraic framework, facilitating the evaluation of multicenter molecular integrals over Slater-Type Orbitals.

The proposed formulations demonstrate their particular strength when dealing with high-order quantum numbers, where conventional numerical methods often encounter stability and speed issues. By reducing the reliance on intricate angular transformations, this Cartesian-based binomial logic provides a robust pathway for faster and more precise molecular simulations. We further discuss the computational advantages of these formulae, highlighting their potential to streamline large-scale quantum chemical calculations and improve the scalability of multicenter integration algorithms

***K*eywords** Legendre functions · Multicenter integrals · Regular solid spherical harmonics

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ONLINE TENSOR TRAIN DECOMPOSITION WITH SEQUENTIAL ORTHOGONALIZATION FOR IN-SITU COMPRESSION OF PDE SOLUTIONS

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ABSTRACT

High-dimensional partial differential equation (PDE) simulations are fundamental to computational science and engineering, yet they generate large volumes of time-series data. This data generation creates memory and storage bottlenecks, as the time required to write data to disk can exceed the computation time itself. While Tensor Train (TT) decomposition [1] offers an effective algebraic framework for mitigating the curse of dimensionality, conventional batch-based TT algorithms require storing the full dataset in memory, making them computationally prohibitive for the in-situ compression of streaming simulation data. Conversely, existing online TT methods [2], which rely on recursive approximations, often suffer from numerical instability, ill-conditioned Gram matrices, and error accumulation over long temporal evolutions. In this work, we propose applying a deterministic online TT decomposition algorithm (Online TT-ALS) for the in-situ compression of streaming PDE solutions. Our method sequentially enforces orthogonal constraints on the TT cores during a single-sweep update. By preserving an exact orthogonal subspace at every time step, our approach completely avoids the inversion of ill-conditioned matrices. This property ensures long-term numerical stability, approximation accuracy without requiring an initial warm-up period, and low-latency processing suitable for dynamic environments. We formulate an online compression pipeline tailored for time-dependent PDEs and investigate its practical feasibility. To validate the algorithm, we conduct numerical experiments on fundamental benchmark models, including the high-dimensional heat equation. Preliminary results indicate that the proposed framework achieves significant data reduction while maintaining mathematical fidelity and computational efficiency. We evaluate the algorithm's temporal stability and compression performance, demonstrating its potential to scale to broader classes of multidimensional PDEs and to provide a robust in-situ compression strategy for scientific computing.

***K*eywords** TT decomposition · PDE simulation · data compression

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A HYBRID OF THE CGS METHOD AND THE GPBI-CGSTAB(L) METHOD FOR NONSYMMETRIC LINEAR SYSTEMS

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ABSTRACT

We focus on numerical methods for nonsymmetric linear systems, in particular on a hybrid variant developed within the framework of product-type Krylov subspace methods [3]. In [2], a hybrid variant of the CGS method and the GPBi-CG method was recently proposed for solving nonsymmetric linear systems. In this talk, we further investigate the possibility of this hybrid variant together with the GPBi-CGstab(L) method, which is a unified algorithm of the GPBi-CG method and the Bi-CGSTAB(ℓ) method [1].

Similar to the approach in [2], we consider a three-phase algorithm in which either the CGS method or the GPBi-CGstab(L) method is applied in each phase. The transitions between phases are controlled by user-defined parameters ϵ_1, ϵ_2. In the first phase, the GPBi-CGstab(L) method is applied. Then, when the relative residual falls below ϵ_1, the algorithm transitions to the second phase, in which the CGS method is employed. Finally, when the relative residual falls below ϵ_2, the algorithm transitions to the third phase, where the GPBi-CGstab(L) method is applied again. To switch to a different method, it can be easily achieved by changing the parameters ζ, η in the framework proposed in [3], while preserving the Krylov subspace information gained in the previous iterations.

Numerical experiments are conducted on three-dimensional convection-diffusion-reaction equations in both diffusion-dominant and convection-dominant settings. The results show that, in the convection-dominant setting, classic stabilized methods such as Bi-CGSTAB and GPBi-CG either break down or stagnate. Although the CGS method converges, it suffers from a significant loss of accuracy. In contrast, our porposed hybrid method achieves fast and robust convergence without a significant loss of accuracy.

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***K*eywords** Nonsymmetric linear systems · Krylov subspace methods · CGS · GPBi-CG · GPBiCGstab(L)

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ACHIEVING LOW VOLTAGE WORKING PYRENE-BASED ORGANIC FIELD-EFFECT TRANSISTOR WITH HIGH MOBILITY

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ABSTRACT

In this study, a series of pyrene-based donor acceptor molecules were synthesized by Suzuki coupling reaction and then characterized by FT-IR and ^{1}HNMR techniques. According to UV-Vis measurements, approximately 100 nm red shift was observed at low energy band after adding extra donor moiety to the pyrene standard molecule. Thus, the optical band gaps were narrowed from 2.12 eV to 1.72 eV. Electrochemical band gap values were also found to be relatively compatible with optical band gap values. The charge distributions at HOMO and LUMO between the donor and acceptor units were demonstrated by theoretical DFT measurements. According to AFM results, rougher surface was observed in the pyrene-TPA due to 3D structure of the TPA subunit on the structure. Finally, the use of a gel form of poly (methy methacrylate) (PMMA) as gate dielectric in pyrene-based organic field effect transistors (OFETs). Gel based dielectric was prepared by adding propylene carbonate into PMMA to increase the total dielectric constants.

Keywords Organic field effect transistors · Pyrene · High mobility

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ACCELERATOR-BASED STRUCTURAL INVESTIGATION OF SEMICONDUCTOR NANOCRYSTALS

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ABSTRACT

This study employs synchrotron-based X-ray absorption spectroscopy (XAFS) to resolve the local elemental arrangements and interfacial bonding within complex colloidal semiconductor heterostructures. By performing multi-edge measurements, we systematically map the atomic distribution across internal boundaries as structural complexity increases.Through coordinated analysis of X-ray absorption near edge structure (XANES) and extended X-ray absorption fine structure (EXAFS) data, we determine critical structural parameters, including interatomic distances (R), coordination numbers (N) and Debye-Waller factors (σ^2). Our results provide evidence of a material gradient at the internal interfaces, revealing that the prevailing interatomic bonds are highly sensitive to the size of the initial core. Furthermore, we address the temporal stability of these systems under ambient conditions. Long-term spectroscopic monitoring over long periods of time indicates that while the internal core remains structurally robust, surface-related bonds undergo partial degradation and atomic reorganization. This research represents a comprehensive experimental assessment of the local atomic order in incrementally heterogeneous quantum-confined systems. The findings offer fundamental insights into how precise elemental arrangements at the nanoscale dictate the electronic and emissive properties of advanced semiconductor materials.

***K*eywords** Synchrothron · XAFS · Colloidal semiconductors

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GENERALISED LEONARDO NUMBERS

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ABSTRACT

In this talk, we provide an overview of the history of Leonardo numbers. We review some of the most recent results concerning this sequence, together with several relevant historical interconnections. We also present a new perspective on these numbers and discuss several conjectures and open problems related to their extensions, particularly those involving modular periodicity.

***K*eywords** Leonardo numbers; recurrence relations

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EXPLICIT GREEN FUNCTION REPRESENTATIONS FOR DIRICHLET PROBLEMS IN COMPLEX POLYDOMAINS

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ABSTRACT

Boundary value problems for complex partial differential equations play a fundamental role in complex analysis and its applications, yet many aspects remain open in the framework of several complex variables. In this work, we investigate the Dirichlet boundary value problem for homogeneous and inhomogeneous complex partial differential equations in polydomains, with particular emphasis on explicit analytic constructions based on Green function representations.

The classical theory of the Dirichlet problem in one complex variable provides powerful tools through integral representations; however, extending these techniques to multidimensional product domains introduces additional analytical difficulties arising from the geometry of polydomain boundaries and the interaction between multiple complex variables. To address these challenges, we develop a Green function approach adapted to polydomains, allowing the derivation of explicit representation formulas for solutions.

Using suitably constructed Green functions, we establish integral representation formulas for holomorphic or complex-valued solutions satisfying prescribed boundary data. These representations provide a unified framework for treating both homogeneous and inhomogeneous equations and lead naturally to necessary and sufficient solvability conditions for the Dirichlet problem. The method highlights the role of boundary symmetries and enables a systematic extension of classical potential-theoretic techniques to higher-dimensional complex settings.

Furthermore, the transition from the unit disc to polydomains is analyzed in detail, demonstrating how multidimensional effects influence the structure of Green kernels and the associated boundary integral operators. Despite the increased complexity, the resulting solvability theory preserves strong analogies with the one-variable case and yields well-posed formulations without imposing excessively restrictive assumptions.

The results contribute to the development of Green function methods in several complex variables and provide a constructive analytical framework for the study of Dirichlet-type boundary value problems in multidimensional complex domains.

Keywords Polydomain · Dirichlet Problem · Green function

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GEOMETRIC STRUCTURE-PRESERVING METHODS FOR TWO-DIMENSIONAL STOCHASTIC NONLINEAR WAVE EQUATIONS

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ABSTRACT

This report proposes a novel structure-preserving numerical framework for two-dimensional stochastic nonlinear wave equations driven by Stratonovich-type multiplicative noise under periodic boundary conditions. Stochastic nonlinear wave equations are foundational mathematical models utilized across various physical systems, including electromagnetics, elasticity, and fluid dynamics, where wave propagation is inevitably influenced by external random perturbations and environmental uncertainties. Since these systems exhibit rich geometric structures within the Hamiltonian framework, conventional numerical methods often introduce unphysical artifacts, artificial energy dissipation, or phase errors during long-time simulations. This motivates the development of structure-preserving numerical schemes capable of accurately capturing the long-time dynamics of stochastic wave propagation.

To preserve the geometric structure, the target equation is reformulated as a multi-symplectic Hamiltonian system with inherent multi-symplectic and momentum conservation properties. While previous literature explored multi-symplectic techniques using radial basis functions [1] or discontinuous Galerkin methods [2], the application of spectral element approaches remains scarce. For spatial semi-discretization, the authors employ the Galerkin spectral element method (GSEM). Drawing inspiration from historical discrete Galerkin projections [3], this approach effectively combines the high accuracy of spectral methods with the geometric flexibility of finite elements.

For temporal discretization, the Crank-Nicolson method is applied. Crucially, a specially designed structure-preserving discretization technique is introduced for the nonlinear terms. This ensures that the resulting fully discrete numerical method rigorously preserves the discrete multi-symplectic structure and conserves momentum. Furthermore, a rigorous convergence analysis is performed in the L^2-norm, establishing an analytical error bound. Finally, two-dimensional numerical experiments validate the theoretical analysis and demonstrate the outstanding capability of the proposed method in maintaining physical invariants over long-time simulations.

Keywords Galerkin spectral element method · Preserving structure · Error estimate

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ON THE CONSTRUCTION AND L^p-CONVERGENCE OF NEW TYPE OF SAMPLING OPERATORS

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ABSTRACT

This study proposes a novel family of sampling operators within the Durrmeyer setting, designated as Hermite-Durrmeyer type operators. Our investigation begins by proving the continuity of these operators within $L^p(\mathbb{R})$ spaces for $1 \leq p < \infty$. We then proceed to analyze the approximation behavior in the norm topology, deriving quantitative error bounds through the use of appropriate moduli of continuity. To substantiate the theoretical framework, we present specific examples of kernels that fulfill the necessary conditions, supported by numerical experiments and graphical representations. The outcomes validate the utility of this new class of operators, specifically illustrating how incorporating derivative data enhances approximation accuracy and verifies the predicted convergence speeds.

Keywords sampling operators · Hermite-Durrmeyer type operators · convergence properties

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DIRECT AND INVERSE RESULTS FOR ITERATED BOOLEAN SUMS OF GENERALIZED SAMPLING OPERATORS

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ABSTRACT

This study analyzes the approximation properties of iterated Boolean sums constructed from generalized sampling operators. We derive a direct theorem concerning the rate of convergence within the uniform norm on the real axis. To demonstrate that this estimate is sharp, we also prove a strong converse inequality. The arguments presented herein are based on established results regarding simultaneous approximation by generalized sampling operators.

***K*eywords** sampling operators · direct results · boolean sums

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FIXED POINT RESULTS ON PERTURBED METRIC SPACES WITH DIRECTED GRAPHS

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ABSTRACT

This study explores the properties of fixed points regarding self-mappings situated within perturbed metric spaces that are equipped with a graph. We derive a Banach-style theorem that guarantees both the existence and uniqueness of fixed points by synthesizing the concepts of perturbed metrics with graph-preserving maps under specific contraction criteria. The findings presented herein effectively extend the scope of numerous established theorems found in both standard and generalized metric literature. Furthermore, numerical examples are included to verify the consistency and potential applications of the proposed theorem.

***K*eywords** fixed point · perturbed metric space · graph theory

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MODELING SEISMIC WAVE PROPAGATION IN TURKIYE USING EXPONENTIAL-TYPE SAMPLING KANTOROVICH OPERATORS

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ABSTRACT

In this talk, we focus on modeling seismic wave propagation in Türkiye using sampling-type operators. Seismic wave models are constructed based on the Local Magnitude (ML) scale derived from data provided by the Turkish National Seismic Network (TNSN), together with a third-order Mellin-spline kernel. The performance of the proposed Kantorovich-type operators is evaluated in comparison with Durrmeyer-type operators. Numerical results and graphical analyses indicate that the Kantorovich-type approach provides lower Mean Square Error (MSE) values and demonstrates superior performance, particularly in the filtering and reconstruction of noisy seismic signals.

***K*eywords** Seismic modeling · Local magnitude

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HYPERBOLIC FEATURE ENGINEERING FOR WIND POWER FORECASTING

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ABSTRACT

Wind power forecasting is typically formulated as a supervised learning problem in which prediction models operate on features constructed in Euclidean spaces. However, when wind power time series are represented through probabilistic models, their parameter spaces naturally inherit non-Euclidean geometric structures. In this study, we propose a hyperbolic feature engineering framework grounded in information geometry for wind power forecasting.

The proposed approach begins by representing local segments of wind power time series via parametric modeling using the Weibull distribution, a widely accepted model for wind-related stochastic processes. When endowed with the Fisher information metric, the Weibull family forms a statistical manifold whose parameter space exhibits constant negative curvature under suitable parameterizations. This intrinsic curvature implies that Euclidean embeddings of the parameters distort the geometric relations induced by the underlying statistical model.

Motivated by this observation, we embed the Weibull-derived statistical representations into a hyperbolic space that is consistent with the manifold's negative curvature. Geometry-aware features are then constructed by exploiting the relational structure induced by the hyperbolic metric, rather than relying on handcrafted descriptors or flat vector operations. The resulting features are integrated into standard forecasting models in a model-agnostic manner.

Experimental evaluations demonstrate that aligning feature representations with the information-geometric structure of the Weibull family improves forecasting performance, particularly in capturing nonlinear variability patterns across wind regimes. The proposed framework establishes a principled connection between statistical manifold theory and feature engineering, offering a mathematically consistent pathway for incorporating non-Euclidean geometry into wind power forecasting.

***K*eywords** Statistical Manifolds · Fisher Information Metric · Hyperbolic Feature Engineering

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SOLVING GENERAL FRACTIONAL OSCILLATION RELAXATION EQUATION BY ROTHE'S APPROACH AND NIM

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ABSTRACT

Abstract Fractional differential equations (FDEs) have gained significant attention due to their ability to model complex physical phenomena involving memory and non-local effects, particularly in fields such as fluid dynamics, biology, and engineering. However, obtaining exact solutions to such equations remains a challenging task, necessitating the development of efficient analytical and numerical methods. In this paper, we investigate a generalized fractional oscillation relaxation equation, also known as the Basset equation, formulated in a Banach space. The equation models the motion of a spherical particle in a viscous fluid under gravitational influence and incorporates fractional derivatives to capture hereditary properties of the system. We establish conditions for the existence and uniqueness of strong and classical solutions. To solve the problem, we develop a novel numerical algorithm by combining Rothe's approach with the New Iterative Method (NIM). The proposed scheme, termed the RN method, is based on temporal discretization using Rothe's technique along with an L1 approximation for the Riemann–Liouville fractional derivative. The resulting formulation transforms the original problem into a system suitable for iterative decomposition. The RN method is constructed as a three-step predictor–corrector scheme, involving two predictor stages followed by a correction stage derived from truncated NIM series expansion. This approach effectively handles the nonlinear term and improves computational efficiency while maintaining accuracy. An illustrative example from fluid dynamics is presented to demonstrate the applicability and effectiveness of the proposed method. The results indicate that the RN method provides a reliable and efficient approximation for solving fractional oscillation relaxation equations. The proposed framework offers a promising tool for analyzing fractional models and can be extended to a broader class of fractional differential equations arising in applied sciences.

***K*eywords** Fractional Differential Equations · New Iterative Method · Rothe's Approach · Basset Equation

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MANNHEIM-LIKE CURVES

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ABSTRACT

In this paper, the concept of a Mannheim curve pair is introduced within the framework of the Frenet-like curve frame in Euclidean 3-space. Necessary and sufficient conditions are established for a polynomial curve to admit a Mannheim partner curve. Several fundamental characterizations of Mannheim partner curves are derived, and the geometric relationships between the corresponding curve pairs are investigated in detail. Furthermore, explicit relations between the curvatures and torsions of the Mannheim partner curves are obtained. These results contribute to the study of special curve pairs in differential geometry.

Keywords First keyword · Second keyword · More but not less than 3

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APPROXIMATION PROPERTIES AND GBS EXTENSIONS OF BIVARIATE (λ, μ)-BERNSTEIN OPERATORS

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ABSTRACT

In this study, we construct a bivariate extension of the (λ, μ)-Bernstein operators recently introduced by Zhou and Cai (2025). We investigate the fundamental approximation properties of these operators, explicitly evaluating the rate of convergence by utilizing both partial and complete moduli of continuity. Furthermore, we examine the approximation errors and the order of convergence through the application of Peetre's K-functional. Beyond the classical operators, we introduce their Generalized Boolean Sum (GBS) variants and analyze their approximation behaviors for Bögel-continuous and Bögel-derivable functions. Finally, we provide comprehensive numerical examples and graphical illustrations to visually demonstrate the theoretical convergence behavior and highlight the operational flexibility offered by varying the parameters λ and μ.

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***K*eywords** (λ, μ)-Bernstein operators · GBS operators · rate of convergence

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TENSOR FORM OF MINARES METHOD FOR GENERALIZED HERMITIAN SYLVESTER QUATERNION TENSOR EQUATION WITH AN APPLICATION

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ABSTRACT

This paper concerns the numerical solution of generalized Hermitian Sylvester quaternion tensor equations and their applications to color video encryption and decryption [1]. A generalized Hermitian Sylvester quaternion tensor equation frequently arises in multidimensional signal and image processing problems. Quaternion tensors effectively represent multidimensional color data [2] by preserving correlations among color channels and structural information in color videos. However, the corresponding quaternion tensor equations are large-scale and computationally expensive to solve, especially when traditional real representation techniques are adopted. Such techniques often destroy the original tensor structure and lead to a significant increase in storage requirements and computational complexity. These issues pose significant challenges for the efficient numerical solution of large-scale quaternion tensor equations.

Motivated by these challenges, we extend the MinAres method [3], which is originally designed for symmetric linear systems, to the generalized Hermitian Sylvester quaternion tensor equation. Using quaternion tensor properties and tensor operations, a tensor form of the MinAres method is developed to solve the considered problems. The proposed method works directly with quaternion tensors, thereby preserving the tensor structure and avoiding dimension expansion. In addition, the proposed method inherits some favorable numerical properties of the classical MinAres method and can be implemented for large-scale problems.

Numerical experiments show that the proposed method achieves comparable residual accuracy with lower computational cost than existing iterative methods. Furthermore, applications to color video encryption and decryption show that the proposed method can recover color video data while preserving the quaternion tensor structure. Comparisons with some existing iterative methods(e.g., the MinRes method [4]) are also provided to further show that the proposed method requires less CPU time and storage while maintaining similar numerical accuracy.

Keywords Quaternion · Hermitain · Sylvester quternion equation · Tensor form · MinAres

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GENERALIZED ANALYTIC FUNCTIONS IN COMPLEX DOMAINS

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ABSTRACT

Generalized analytic functions constitute an important extension of classical holomorphic functions and provide a powerful framework for the study of elliptic partial differential equations. In this work, boundary value problems for generalized analytic functions are investigated by employing methods from complex analysis and the theory of partial complex differential equations. The study begins with the analysis of inhomogeneous Cauchy–Riemann systems and the derivation of integral representations for their solutions through complex integral operators. These representations are then used to establish connections between holomorphic functions and solutions of more general complex differential equations. Particular attention is devoted to the Dirichlet problem for the first-order equation

$$\frac{\partial w}{\partial \bar{z}} + \lambda w = 0,$$

in bounded simply connected domains with Hölder continuous boundary data. By transforming the problem into an operator equation and applying the contraction mapping principle, existence and uniqueness results are obtained under suitable conditions on the parameter λ. Furthermore, explicit estimates for the corresponding solutions are derived in Hölder spaces. The developed approach is subsequently extended to a second-order partial complex differential equation through its reduction to an associated first-order system. This reduction allows the application of fixed-point techniques and complex integral operator methods to establish solvability results and derive a priori bounds for the solutions. The results demonstrate how classical tools of complex function theory can be effectively combined with functional analytic methods to study boundary value problems for generalized analytic functions and related elliptic equations. The obtained estimates provide a theoretical basis for further investigations of higher-order complex differential equations and their boundary value problems.

***K*eywords** Generalized analytic functions · Dirichlet problem · Cauchy–Riemann equations · complex integral operators

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EULER WAVELETS SCHEME FOR NEUTRAL FRACTIONAL DELAY DIFFERENTIAL EQUATION

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ABSTRACT

This article comprehensively studies the Neutral Fractional Volterra Delay Integro-Differential Equation (NFVDIDE) with variable coefficients and weakly-singular kernel. It takes into account Caputo's fractional derivative of orders $\alpha \in (0, 1]$ and $\beta \in (0, \alpha]$ for the current state and delayed state, respectively. To establish the existence results, we utilized the weighted norm concept and the Banach Contraction Principle. Furthermore, we established the Ulam-Hyers stability result. Additionally, we proposed an interpolationbased robust numerical algorithm to approximate the solutions of NFVDIDE and conducted a detailed error analysis. The proposed algorithm achieved a computational accuracy of $O(h^{3-\alpha})$. We analyzed several examples to demonstrate our theoretical findings and illustrated the efficacy of the obtained numerical scheme for various values of α, β.

Keywords Fractional Derivative · Delay · Euler Fractional Wavelets

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SYNTHETIC TABULAR DATA GENERATION UNDER PHYSICAL CONSTRAINTS FOR PHASE CLASSIFICATION AND ALLOY RECOMMENDATION IN DATA-SCARCE HEA STUDIES

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ABSTRACT

We present a mathematically grounded workflow to alleviate data scarcity in High-Entropy Alloys (HEAs) by generating, filtering, and exploiting synthetic tabular data for downstream tasks in phase classification and candidate recommendation. The approach integrates four families of generators—CTGAN, TVAE, Gaussian copulas, and SMOTE—evaluated through a triple protocol that balances statistical fidelity (SDMetrics column-shape and pair-trend scores), predictive utility via TSTR (Train Synthetic, Test Real), and novelty/privacy through distance-to-closest-record (DCR). We cast the amount and mixture of synthetic data as an optimization problem, applying Bayesian search (Optuna) with early stopping to identify generator proportions that maximize weighted F1 over Random Forest and XGBoost while preserving plausibility.

At the sample level, we introduce an individual quality score combining a KDE-based likelihood (fidelity), outlier penalties (Isolation Forest), and a target "sensible novelty" term shaped on DCR. Percentile-based filtering shifts solutions toward a Pareto-efficient frontier between fidelity and predictive performance. Beyond augmentation, we deploy a TVAE-driven exploratory stage for alloy recommendation: generate a virtual composition library under compositional constraints, screen with a calibrated phase classifier, score by a target novelty function that favors moderate distance from the real support, and select diverse representatives via clustering. A lightweight thermodynamic post-filter using $\Delta Hmix$, $\Delta Smix$, δ, Ω, and VEC further prioritizes physically plausible candidates.

On a curated HEA dataset, the best configuration reaches a mean F1 of 0.814 in the full-data scenario and 0.777 when starting from 25% of real data. Under exploratory generation, 3,288 high-confidence BCC/FCC candidates are proposed, of which 1,555 pass the thermodynamic screen. Validation across external datasets indicates that gains are largest when real data are scarce and structurally coherent; improvements attenuate on noisier bibliographic corpora, underscoring data quality as a moderator of augmentation benefits. The contribution is a reproducible, optimization-driven pipeline that formalizes the augmentation–fidelity trade-off, integrates domain constraints, and supports early-stage discovery via diverse, plausibly novel compositions.

***K*eywords** Synthetic tabular data · CTGAN · TVAE · Gaussian copulas · SMOTE · Bayesian optimization · TSTR · DCR · HEAs · phase classification · alloy recommendation · thermodynamic screening

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MODULAR HAUSDORFF INTERPOLATIVE METRIC REGULARITY AND COINCIDENCE PROBLEMS WITH APPLICATIONS

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ABSTRACT

In this work, we introduce and explore the innovative geometric framework of "Modular Hausdorff Interpolative Metric Spaces", which effectively unifies the scale-dependent structural flexibility of modular metric spaces with the non-linear, multiplicative error-bounds inherent in interpolative metric geometries. Within this newly established topological setting, we formulate the novel concept of "Modular Hausdorff Interpolative Metric Regularity" for multivalued operators. By analyzing the behavior of the modular open covering rate under interpolative constraints, we establish several robust coincidence point theorems and existence criteria for generalized set-valued inclusions. Furthermore, leveraging the parametric control provided by the modular scale parameter λ, we investigate the sensitivity and robustness of the solution sets, yielding comprehensive Ulam-Hyers stability results that extend classical linear variants. The theoretical framework developed herein provides powerful new variational tools capable of characterizing equilibrium dynamics and stability profiles in systems where classical metric regularities fail due to singularity or unboundedness. To demonstrate the analytical potency of our abstract formulations, we present an application to the existence and stability of solutions for a class of nonlinear implicit integral inclusions.

***K*eywords** multivalued operators · modular metric spaces · nonlinear contraction · interpolative metric spaces · metric regularity

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BICOMPLEX MANDELBAR DYNAMICS ASSOCIATED WITH THREE CONJUGATIONS

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ABSTRACT

In this paper, we investigate the antiholomorphic counterpart of bicomplex quadratic dynamics by defining and analyzing bicomplex Mandelbar sets corresponding to the three natural involutive conjugations of the bicomplex algebra. For each conjugation $\dagger_m$, $m \in 1, 2, 3$, we study the iteration

$$F_{c,\dagger_m}(\eta) = (\eta^{\dagger_m})^2 + c$$

on $\mathbb{C}_2$ and define the associated parameter set via the boundedness of the orbit of the origin. Our main objective is to determine how the choice of bicomplex conjugation influences the structure of the resulting antiholomorphic parameter space. Using the idempotent decomposition of bicomplex numbers, we obtain a conjugation-dependent classification of the corresponding dynamics. The conjugation $\dagger_3$ yields a genuine Cartesian product of two classical Mandelbar sets, whereas $\dagger_2$ generates a coupled two-parameter quartic system. In contrast, $\dagger_1$ produces a conjugate-dependent coupling between the idempotent components and therefore does not admit a Cartesian-product representation. We also introduce three-dimensional slices of these fractal sets for visualizations and investigate their reflection symmetries. Theoretical results and graphics demonstrate that the three conjugations generate geometrically distinct fractal structures despite sharing certain symmetry properties. These results clarify the role of bicomplex conjugations in antiholomorphic dynamics and show that bicomplex Mandelbar theory exhibits structural phenomena that do not arise in the holomorphic bicomplex Mandelbrot setting.

Keywords Bicomplex numbers · Mandelbar set · antiholomorphic dynamics · idempotent decomposition · bicomplex conjugations · fractal visualization.

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PERTURBED MODULAR HAUSDORFF INTERPOLATIVE IFS AND FRACTAL ATTRACTOR STABILITY

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ABSTRACT

This study presents a modular Hausdorff interpolative approach to fractal attractors. In contrast to the classical Hutchinson construction, the ordinary Hausdorff distance is replaced by a scale-dependent modular Hausdorff distance. For a finite family of modular contraction mappings $T_1, T_2, \ldots, T_N$, we study the modular Hutchinson operator

$$\mathcal{F}(A) = \bigcup_{i=1}^{N} T_i(A).$$

Under appropriate completeness and contractive assumptions, we prove the existence and uniqueness of a compact attractor $\mathcal{A}^*$ that satisfies $\mathcal{F}(\mathcal{A}^*) = \mathcal{A}^*$. Moreover, the Picard iteration generated by $\mathcal{F}$ converges to this attractor for every admissible initial compact set.

The second part of the study deals with perturbation stability. We explain that small modular Hausdorff perturbations of the Hutchinson operator lead to controlled deviations between the corresponding attractors. This gives a scale-dependent stability estimate for fractal generation and provides a modular extension of the classical fractal attractor theory. A Sierpiński-type modular construction is included to illustrate the applicability of the result.

Keywords modular metric space · Hausdorff interpolative metric · Hutchinson operator · iterated function system · fractal attractor · stability

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GPBiCG(m, ℓ, k): An Extension of GPBiCG(m, ℓ) with Preliminary Adaptive Parameter Selection Strategies

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Abstract

We propose a new Krylov subspace method, denoted GPBiCG(m, ℓ, k), for solving large sparse non-Hermitian linear systems. The method combines features of BiCGSTAB [2], GPBiCG [3], and LOPBiCG [4] within a unified framework. Each cycle consists of m iterations of BiCGSTAB, followed by ℓ iterations of GPBiCG and k iterations of LOPBiCG. The proposed algorithm extends the GPBiCG(m, ℓ) method of Fujino [1] by introducing an additional parameter that increases the flexibility of the residual polynomial construction.

Numerical experiments are performed on a collection of benchmark sparse matrices arising from scientific and engineering applications. The influence of the parameters m, ℓ, and k on convergence behavior is investigated, and the performance of GPBiCG(m, ℓ, k) is compared with BiCGSTAB, BiCGSTAB2, GPBiCG, and GPBiCG(m, ℓ). The results show that, for suitably selected parameter combinations, GPBiCG(m, ℓ, k) can achieve faster convergence and improved robustness on several test problems.

Because the effectiveness of the method depends strongly on the parameter choice, we also examine preliminary adaptive parameter-selection strategies based on cumulative residual reduction criteria. Although these strategies can identify effective parameter combinations in some cases, their performance remains sensitive to the initial parameter settings and does not yet consistently reproduce the best fixed configurations obtained through offline parameter selection.

These results indicate that GPBiCG(m, ℓ, k) provides a promising extension of existing product-type Krylov subspace methods. At the same time, the development of reliable and fully automatic parameter-selection procedures remains an important topic for future research.

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***K*eywords** Non-Hermitian linear systems · Krylov subspace methods · GPBiCG · Residual polynomials · Parameter selection strategies

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A HIGH–ORDER ACCURATE ALE–SBP NUMERICAL SCHEME TO MODEL EVAPORATION

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ABSTRACT

We present a high–order numerical method for the canonical Stefan and sucking interface evaporation problems [1]. The governing energy equation is formulated within the Arbitrary Lagrangian–Eulerian (ALE) framework [2], which provides a systematic way to handle evolving computational domains. To avoid element collapse, the moving mesh is mapped onto a fixed reference domain through a coordinate transformation [3]. The transformed model is discretized using the continuous Galerkin (CG) finite element method (FEM) within the Summation–By–Parts (SBP) framework [4]. The SBP construction ensures discrete stability by mimicking continuous integration–by–parts identities. Numerical experiments are carried out with polynomial orders ranging from 1 to 4, allowing systematic evaluation of convergence behavior, accuracy, and efficiency. These experiments demonstrate the importance of high–order discretizations in capturing interface dynamics with reduced computational cost. In addition, employing high–order, provably stable discretizations ensures that reliable solutions to evaporation problems can be obtained with very small errors. Beyond benchmarking, the proposed ALE–FEM–SBP scheme establishes a foundation for robust simulation of phase–change phenomena, offering a provably stable and high–order accurate approach that can be extended to more complex geometries and multiphysics settings.

***K*eywords** Evaporation · Stefan problem · sucking problem · summation–by–parts · continuous Galerkin method

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NEW FIXED POINT RESULTS FOR GENERALIZED (ψ, θ, L) -TYPE $\mathcal{R}$ - CONTRACTIVE MAPPINGS IN $\mathcal{F}$-METRIC SPACES WITH AN APPLICATION

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ABSTRACT

The goal of this paper states a fixed point theorem for generalized (ψ, θ, L)-type $\mathcal{R}$-contractive mappings in $\mathcal{F}$-metric spaces having a binary relation. This analysis is based on a relation theoretic approach, where the condition of contraction needs to hold only for those elements that are related by the relation $\mathcal{R}$. The main condition of contraction can be described by means of an altering distance function, a control function and an extra perturbation term with $N_T(x, y)$. As the classical triangle inequality does not hold in $\mathcal{F}$-metric spaces, a Picard admissible condition is defined along with the Picard orbit to cope with the mixed distance term in $M_T(x, y)$. The existence of fixed point under conditions of $\mathcal{R}$ -completeness, (T)-closedness of the relation $\mathcal{R}$, $\mathcal{R}$ -continuity, and an appropriate condition for the auxiliary function is demonstrated. A result on the uniqueness of the fixed point is obtained under the comparability of fixed points through the binary relation. This theorem includes Banach-type and generalized $\mathcal{R}$ -contractive type results. As an application, the proposed fixed point theorem is used to study a Caputo-type fractional initial value problem. The fractional differential equation is reformulated as an equivalent integral equation, and the corresponding integral operator is examined using the existing fixed point methodology. The necessary conditions for the existence and uniqueness of a solution are determined. Moreover, an Ulam–Hyers stability result is achieved under a Lipschitz-type assumption. The example shows how relation-theoretic fixed point methods in $\mathcal{F}$-metric spaces can be utilized to study the stability analysis of nonlinear fractional models.

***K*eywords** Fixed point; $\mathcal{F}$-metric space; binary relation;$\mathcal{R}$ -contractive mapping; fractional differential equation; Ulam–Hyers stability

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VARIATIONAL QUANTUM ALGORITHMS: MACHINE LEARNING, MATHEMATICAL FOUNDATIONS AND IMPLEMENTATIONS

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ABSTRACT

Variational Quantum Algorithms (VQAs) have evolved as a top contender in the reality of quantum computation, considering the Noisy Intermediate-Scale Quantum (NISQ) phase [Bayad, KS]. The optimization problems can be solved based on shallow quantum circuits, with the help of classical numerical routines, by the virtue of their hybrid quantum classical structure [GGL]. This paper will provide an in-depth discussion of VQAs in machine learning with a special focus on the mathematical basis of expressivity, optimization geometry, and convergence behaviour. With the assistance of the tools of functional analysis, operator theory, and quantum information geometry, we derive the structure of parameterized quantum circuits and examine the landscape characteristics of variational loss functions. We also apply a number of variational quantum machine learning models, the Variational Quantum Classifier, and Quantum Neural Networks, to simulate quantum processors. Numerical experiments indicate that VQAs can be competitive in smaller-scale classification tasks but that noise, the structure of the ansatz and the stability of optimization have a significant impact on performance. The findings demonstrate the potential and constraints of VQAs in the available quantum technologies and inform future algorithm and hardware development [Simsek2019, Simsek2021].

***K*eywords** Variational Quantum Algorithms · Quantum Machine Learning · Parameterized Quantum Circuits · Hybrid Quantum–Classical Optimization

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MATHEMATICAL CHARACTERIZATION OF ACCURACY LOSS IN DIFFERENTIALLY PRIVATE MACHINE LEARNING MODELS

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ABSTRACT

Differential Privacy is a theoretical framework for ensuring data privacy when conducting statistical analysis. It was introduced to quantify privacy in statistical analyses. However, as we progress and machine learning takes over, it is essential to assess our progress in privacy as well. . Initially, DP algorithms focused on sanitising simple statistics, but in recent years, it has made its way to machine learning. There have been several advancements in the basic differential privacy models over the years to combat with changes. While Rényi Differential Privacy (RDP) has been widely adopted due to its composability properties, Bayesian Differential has emerged as an alternative that adapts noise based on posterior distributions. In this study, I analyse the mathematical properties of accuracy loss under these two DP frameworks and explore their implications for differentially private machine learning models. I theoretically analyse the mathematical bounds for accuracy tests and compare how RDP and Bayesian DP affect model performance. To support this analysis, I conduct empirical experiments on differentially private logistic regression models and evaluate accuracy loss across different privacy budgets. To verify the theorem, I implement and test the bounds on MATLAB, providing a visual and intuitive understanding. This research provides insights into the privacy-utility trade-offs of these DP mechanisms and a look into the better algorithm supported by theoretical bounds.

***K*eywords** Differential Privacy · Bayesian Differential Privacy · Privacy Preserving Machine Learning · Renyi Differential Privacy

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ERROR CONTROL TECHNIQUE OF QUADRATURE-BASED ALGORITHMS FOR THE VON NEUMANN ENTROPY

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ABSTRACT

This study considers a quadrature-based method for computing the von Neumann entropy. For a positive semidefinite Hermitian matrix $A \in \mathbb{C}^{n\times n}$, the von Neumann entropy is defined as $\mathrm{tr}(-A\log(A))$, where $\mathrm{tr}(\cdot)$ denotes the trace (the sum of diagonal entries of a square matrix) and $\log(A)$ denotes the matrix logarithm. In numerical computation, we require evaluating the quadratic form $\boldsymbol{b}^\top(-A\log(A))\boldsymbol{b}$ for a vector $\boldsymbol{b} \in \mathbb{R}^n$. The computation of an integral representation of $\boldsymbol{b}^\top(-A\log(A))\boldsymbol{b}$ is reduced to solving dozens or hundreds of shifted linear systems. Current approaches usually analyze the quadrature discretization error [1], but rarely take into account the additional error introduced by solving these shifted linear systems with iterative solvers.

In this study, we bound this error using the residuals of the approximate solutions of these linear systems. This yields a stopping criterion for iterative solvers that keeps the error in $\boldsymbol{b}^\top(-A\log(A))\boldsymbol{b}$ below a prescribed tolerance. Numerical results demonstrate that the proposed criterion enables the computation of $\boldsymbol{b}^\top(-A\log(A))\boldsymbol{b}$ within the prescribed tolerance.

***K*eywords** matrix function · von Neumann entropy · numerical quadrature

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A Fractional-Order Model of Adenovirus Infection and Oncolytic Cancer Therapy: Stability, Bifurcation, and Optimal Control

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Abstract

Adenoviruses present a unique challenge in modern medicine due to their dual role as both infectious pathogens and promising agents in oncolytic cancer therapy. While adenovirus infections can spread within populations and cause significant health concerns, engineered adenoviruses have also been shown to selectively target tumor cells and stimulate antitumor immune responses. Motivated by this dual nature, we develop a novel fractional-order compartmental model that simultaneously describes adenovirus transmission dynamics and its therapeutic interaction with cancer. To the best of our knowledge, this study presents the first fractional-order framework that simultaneously captures adenovirus transmission dynamics and its oncolytic therapeutic effects within a unified population-level model. The proposed model divides the population into five compartments: healthy susceptible individuals, cancer patients susceptible to infection, infected individuals, recovered individuals with temporary immunity, and cancer patients in therapeutic remission induced by viral activity. The model is formulated using Caputo fractional derivatives to account for memory effects and hereditary properties that are commonly observed in biological and immunological systems. Theoretical analysis establishes the positivity and boundedness of solutions and derives the disease-free equilibrium and the basic reproduction number. Using the next-generation matrix approach and fractional stability theory, we show that the disease-free equilibrium is locally asymptotically stable when the basic reproduction number is less than one and unstable when it is greater than one. Furthermore, the model exhibits the possibility of backward bifurcation, indicating that adenovirus persistence may occur even when the reproduction number is below unity. This result reveals a more complex disease-control scenario than that predicted by classical epidemic models. A sensitivity analysis identifies the transmission rate, recovery rate, and increased susceptibility of cancer patients as the most influential parameters governing disease dynamics. To investigate effective intervention strategies, an optimal control framework is formulated by incorporating preventive measures, enhanced virotherapy, and immunotherapy. The resulting optimality system is derived using Pontryagin's Maximum Principle. The optimal strategy suggests that combining preventive measures with therapeutic interventions yields the most effective outcomes. Numerical simulations support the analytical findings and demonstrate that combining preventive interventions with therapeutic strategies can substantially reduce infection prevalence while increasing cancer remission outcomes. The proposed framework provides a new perspective on the interaction between infectious disease dynamics and cancer treatment and highlights the potential of fractional-order modeling in analyzing complex biomedical systems.

Keywords Adenovirus Dynamics, Fractional-Order Modeling, Oncolytic Virotherapy, Cancer Immunotherapy, Bifurcation Analysis, Optimal Control

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ENTROPY GENERATION OF AN MHD JEFFREY NANOFLUID FLOW WITH NON-LINEAR HEAT FLUX: APPLICATIONS IN SOLAR ENERGY

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ABSTRACT

The global requirement for sustainable energy supply to enhance industrial productivity and reduce production costs has caught the attention of researchers on renewable energy in recent years [1]. Solar energy mitigates the dangers associated with the deployment of fossil fuels in the generation of electricity [2]. This research investigates the entropy generation analysis and heat transfer enhancement of magnetohydrodynamic (MHD) Jeffrey nanofluid flow under the influence of nonlinear thermal radiation, heat source, and Cattaneo–Christov heat flux model. The governing partial differential equations of the models are formulated with the aid of conservation laws and transformed into non-dimensional, coupled nonlinear ordinary differential equations using suitable similarity transformations. The resulting non-dimensional models are solved using the spectral local linearization method [3, 4]. The accuracy and the convergence of the method are shown by comparing the obtained results with a known work in the literature, and they are observed to be in good agreement. The impacts of the emerging non-dimensional parameters on the fluid flow profiles are discussed graphically and in tabular form. The results show that the temperature of the fluid increases with the rise in the heat source and thermal radiation parameters. The velocity profile and the magnetic field parameter exhibit an inverse relationship, whereas the temperature of the fluid shows an opposite impact. The findings of this study will shed more light on the design and improvement of solar energy systems, thermal energy technologies, and various electronic cooling systems.

***K*eywords** Jeffrey fluid · Nanofluid · Entropy Generation · Solar Energy · Cattaneo-Christov heat

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REQUIRED NUMBER OF OBSERVATIONS FOR ACCURATE STATE ESTIMATION IN THE LORENZ96 MODEL UNDER DIFFERENT FORCING PARAMETERS

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ABSTRACT

Data assimilation is a method to improve the accuracy of state estimation by combining model predictions with observation data. However, observing all state variables is infeasible in many high-dimensional systems, such as those encountered in weather forecasting and ocean modeling applications. Consequently, for a given dynamical system, the accuracy of state estimation depends on the choice of the observation operator. Previous studies have established theoretical and numerical results for data assimilation in partially observed chaotic dynamical systems [1, 2]. However, conventional theoretical analyses often consider fixed observation operators that do not account for the time-varying instability of the underlying dynamics. To address this limitation, adaptive observation strategies have been proposed [3], which target the most unstable directions of the dynamics at each time step. These strategies can reduce the number of observations required for accurate state estimation. However, it remains unclear how their performance varies with the degree of instability of the underlying dynamics.

In this work, as a typical example of data assimilation for chaotic dynamical systems, we consider the three-dimensional variational (3DVAR) method applied to the Lorenz96 model [4]. Within this framework, we investigate adaptive-in-time observation strategies under different values of the forcing parameter F, which affects the instability and complexity of the dynamics. Numerical experiments are conducted to examine how the minimum number of observations required for accurate state estimation depends on the instability of the dynamics and the observation error. Our numerical results show that adaptive observations remain effective across a range of values of F, while the required number of observations increases with the instability of the dynamics. These results provide useful numerical evidence for understanding the performance of adaptive observation methods and may support future theoretical developments in adaptive observation strategies in data assimilation.

***K*eywords** Data assimilation · Chaotic system · Dynamical instability · 3DVAR · Adaptive observations · Filter accuracy

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AUGMENTED MESH REFINEMENT IN PHYSICS-INFORMED NEURAL NETWORKS FOR EFFICIENT SOLUTION OF STIFF PARTIAL DIFFERENTIAL EQUATIONS

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ABSTRACT

Physics-Informed Neural Networks (PINNs) have emerged as a promising approach for solving partial differential equations; however, their performance often degrades when applied to stiff PDEs due to the inability of uniform training meshes to accurately capture localized sharp gradients without incurring high computational cost. This limitation leads to inefficient learning and slow convergence, particularly in regions with highly nonlinear behavior. To address this issue, this study proposes an Augmented Mesh Refinement Physics-Informed Neural Network (Aug-MR PINN), a novel framework that dynamically reallocates training points toward regions with high residual errors. The proposed method employs residual-based criteria to adaptively refine the mesh during training, enabling the model to focus on critical regions while avoiding unnecessary computation in smoother areas. This adaptive strategy enhances the representation of localized features and improves overall training efficiency. Experimental results demonstrate that Aug-MR PINN consistently achieves lower loss values and faster convergence compared to existing approaches, including AMR-FEM, standard PINN, R-PINN, and X-PINN. Furthermore, the proposed refinement strategies exhibit comparable performance, highlighting the robustness and flexibility of the framework.Overall, this work provides an effective and scalable solution for solving stiff PDEs and offers strong potential for broader applications in computational physics, including fluid dynamics, wave propagation, and multiphase systems.

***K*eywords** Mesh refinement · Physics Informed Neural Networks · Stiff PDE · Mesh grid

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FINITELY-VALUED MODAL LOGICS THAT CANNOT DISTINGUISH SERIAL FRAMES

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ABSTRACT

Badia, Caicedo and Noguera [1] proved that every class of crisp Kripke frames definable by validity in a modal logic valued in a finite lattice algebra is definable in classical modal logic, and that the converse holds whenever the algebra interprets a Boolean algebra through a unary term. Whether this Boolean-interpretation hypothesis is necessary was left as a central open problem. This talk presents a broad negative answer for natural families of many-valued algebras. The main tool is a collapse pre-order: a pre-order compatible with all basic operations, possessing a least element, and placing no element above the top truth value. For any algebra carrying such a pre-order, validity of a modal formula on a frame reduces to its value under a single least valuation. On serial Kripke frames, this value is independent of the frame. Consequently, these logics cannot distinguish any two serial frames, although classical modal logic can separate standard serial-frame subclasses, including reflexive, transitive, and symmetric frames. The construction applies to all order-preserving finite lattice algebras, including negation-free reducts of finite Łukasiewicz chains, and to non-Boolean De Morgan chains. The Belnap–Dunn four-valued algebra does not admit a collapse pre-order, but a twist-structure argument yields the same blindness to serial frames. For this algebra, and for every nontrivial monotone bound-preserving algebra with the bounds in its signature, the definable frame classes are exactly those determined by variable-free classical modal formulas. I also explain how an incorrect identity used in the literature for finite MV-algebras can be repaired without changing the earlier conclusions. These results show that some mechanism for recovering two-valued reasoning is indispensable for full classical frame definability, while leaving open whether Boolean interpretability gives the exact boundary.

***K*eywords** Many-valued modal logic · Frame definability · Finite lattice algebra · Kripke frames · Algebraic logic

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HIGH-RESOLUTION BRAINSTEM MRI RECONSTRUCTION AND ANATOMICAL PARCELLATION VIA ADVANCED SAMPLING OPERATORS

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ABSTRACT

High-resolution Magnetic Resonance Imaging (MRI) of the human brainstem is indispensable for the early detection and clinical tracking of neurodegenerative disorders. However, due to its deep anatomical location and susceptibility to physiological motion artifacts, reconstructing raw MR signals and parcellating its complex subcortical structures often introduce severe geometric blurring and pixel-level noise. To overcome these limitations at the software level, this study introduces a rigorous mathematical framework utilizing specialized sampling operators and exponential sampling series to optimize image reconstruction and structural boundary definition.

In this presentation, we focus on leveraging these advanced sampling operators to develop robust, edge-preserving algorithms that suppress noise while maintaining structural connectivity during brainstem MR image reconstruction. We will demonstrate how these sampling-based workflows can enhance the anatomical and functional parcellation of small brainstem nuclei, preserving delicate geometric features without requiring hardware alterations. Furthermore, we discuss the translational potential of this framework into clinical practice, showing how precise operator-driven feature extraction can serve as a non-invasive quantitative biomarker to assist in early-stage neuro-diagnostics. By bridging pure approximation theory with advanced neuroimaging data processing, this framework establishes a scalable computational foundation to improve the precision of clinical brainstem health assessments.

***K*eywords** Sampling Operators · Brainstem MRI

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A Digital Topology Framework for High-Resolution Brainstem MRI Reconstruction, Automated Segmentation, and Clinical Neuro-Diagnostics

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Abstract

High-resolution Magnetic Resonance Imaging (MRI) of the human brainstem is critical for the early diagnosis and tracking of neurodegenerative and psychiatric disorders. However, due to its small size, deep anatomical location, and susceptibility to physiological motion artifacts, reconstructing and segmenting brainstem structures often introduces geometric distortions and pixel-level noise that can obscure vital clinical pathways. To address these computational challenges, this study introduces a novel image processing framework based on digital topology to enhance multi-contrast MRI reconstruction and structural connectivity analysis.

In this presentation, we focus on leveraging digital metric spaces and topological data analysis to establish robust, connectivity-preserving algorithms for brainstem MR image reconstruction at the software level. We will demonstrate how these topological invariants can be integrated into automated image post-processing workflows to achieve high-fidelity segmentation of complex subcortical nuclei and maintain topological consistency during multi-modal data standardization. Furthermore, we discuss the clinical translation of this framework, showing how topological feature extraction can serve as a non-invasive, quantitative biomarker to detect subtle structural alterations in clinical neuro-diagnostics. By bridging pure mathematical topology with advanced neuroimaging and clinical practice, this framework provides a scalable computational foundation to enhance the precision of brainstem health assessments.

***K*eywords** Digital Topology · Brainstem MRI

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MACKAY'S APERIODIC TILING

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ABSTRACT

In this talk we shall describe the historical context and the basic properties of a relatively unknown early aperiodic tiling with two prototiles. It was presented by the British crystallographer Alan Lindsay Mackay (1926–2025) in 1975 [4], when he described his ideas on the possibility of highly regular structures, but not periodic like "normal" crystals — at that time only a conjecture, but in 1982 proven to be possible by the discovery of quasicrystals [5]. Mackay's tiling was intended as a model of a two-dimensional aperiodic hierarchical structure based on regular pentagons. On a first glance his construction has an infinite set of prototiles of two types: congruent regular pentagons, and golden triangles unlimitedly increasing in size. However, Mackay provided a graphical hint of substitution rules (for regular pentagons and golden triangles), that suggests that the same pattern is obtainable using two prototiles: a golden triangle, and a golden gnomon. He never published the details of his construction, but his method indeed results in an aperiodic substitution tiling of the plane using only two prototiles [2, 3, 4, 5].

Here we shall give the historical context of Mackay's discovery, discuss the reasons why Mackay's tiling until now received little attention in mathematical literature on aperiodic tilings, and provide arguments that his discovery was independent from the other more famous aperiodic sets with two tiles (Penrose's, Robinson's, ...) that were discovered at about the same time [2, 3]. Furthermore, we complete his construction with a proof that his tiling is indeed always an aperiodic substitution tiling with two prototiles, we describe the necessary but originally missing substitution rules for golden gnomons, and show that all such tilings have ϕ^2 as inflation factor [1].

Keywords aperiodic tilings · history of geometry · quasicrystals · Alan Lindsay Mackay · mathematical crystallography

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A SPATIAL NONLINEAR MATHEMATICAL MODEL ON THE IMPACT OF VECTOR CONTROL STRATEGIES AND THE DYNAMICS OF MALARIA TRANSMISSION

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ABSTRACT

Malaria is a life-threaten illness with negative effects on both the social, and economic aspects of human life. Researching into its curtailment or eradication is necessary for elevating human health and social-economic status. In this regard, this study focuses on the spatial nonlinear mathematical model to investigate how vector control strategies are correlated with the dynamics of malaria transmission. The model employs nonlinear partial differential equations (NPDE) to investigate disease transmission. The model system incorporates both human (host) and mosquito (vector) populations. Some applicable epidemiological mathematical analyses were carried out on the model system such as critical points, stability, the basic reproduction number, local asymptotic stability (LAS), global asymptotic stability (GAS), wave speed, and numerical analyses using relevant data were extensively analysed. Using the sharp threshold conditions imposed on the reproduction number, we were able to show that the model exhibited the backward bifurcation phenomenon and the DFE was shown to be globally asymptotic stable (GAS) under certain conditions. It was found that invasive plants have significant effects on malaria transmission. The study suggests that mosquito repellent plants should be planted around the human environment to replace the invasive plants so as to reduce mosquito shelters and feeding opportunities for mosquitoes.

Keywords Spatial model · invasive plants · global stability · traveling wave

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ANALYTICAL CHARACTERIZATION OF FIXED-POINT LOCI IN HIGGS BUNDLE MODULI SPACES VIA L^2-BOUNDS AND LAGRANGIAN GEOMETRY

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ABSTRACT

This work addresses the geometric study of involutions acting on moduli spaces of Higgs bundles over Riemann surfaces, with particular focus on the hyperelliptic involution in the genus-2 case. Higgs bundles, introduced by Hitchin in his seminal work [3], consist of pairs (V, Φ) formed by a holomorphic vector bundle V and a Higgs field $\Phi \in H^0(\Sigma, \mathrm{End}(V) \otimes K_\Sigma)$ satisfying semistability conditions. The moduli space $\mathcal{M}(r, d)$ of semistable rank-r degree-d Higgs bundles carries a natural hyperkähler structure, and automorphisms of the underlying curve induce automorphisms of this moduli space by taking the pull-back [1, 2].

Every genus-2 Riemann surface Σ is hyperelliptic and admits a canonical involution $\iota\colon \Sigma \to \Sigma$ characterized by a degree-2 map $\pi\colon \Sigma \to \mathbb{P}^1$ to the Riemann sphere, branched at six Weierstrass points. This involution can be explicitly described in the affine model $\Sigma = \{(x, y) \in \mathbb{C}^2 : y^2 = f(x)\}$ where $f(x) = \prod_{i=1}^{6}(x - \lambda_i)$, as $\iota(x, y) = (x, -y)$. The hyperelliptic involution induces an automorphism $\iota^*\colon \mathcal{M}(r, d) \to \mathcal{M}(r, d)$ by pullback of both the vector bundle and the Higgs field. The general theory of real structures on Higgs bundle moduli spaces, developed by Schaposnik [5], establishes that the fixed-point locus $\mathcal{M}(r, d)^\iota$ of such involutions forms a Lagrangian submanifold with respect to the natural symplectic structure on the moduli space.

Building upon this result, our main contribution provides explicit quantitative bounds relating the hyperkähler distance between a Higgs bundle and its pullback to analytical invariants of the Higgs field. We focus on the moduli space $\mathcal{M}(2, 0)$ of rank-2 degree-0 semistable Higgs bundles over genus-2 curves, which forms a quasi-projective variety by the work of Nitsure [4]. Specifically, we establish two-sided inequalities of the form

$$\|\Delta(\Phi)\|_{L^2} \leq d_{\mathrm{hyp}}((V, \Phi), \iota^*(V, \Phi)) \leq C \cdot \|\Delta(\Phi)\|_{L^2},$$

where $\Delta(\Phi) = \Phi - \sigma^{-1} \circ \iota^* \Phi \circ \sigma$ represents the difference between the Higgs field and its transformed version under the involution, with $\sigma\colon V \to \iota^* V$ being an isomorphism identifying the bundle with its pullback. The universal constant $C = \sqrt{1 + K^2}$ is determined by the spectral properties of the linearized Hitchin equations. These bounds provide a concrete analytical criterion for measuring proximity to the fixed-point locus, establishing that a Higgs bundle lies in $\mathcal{M}(2, 0)^\iota$ if and only if $\|\Delta(\Phi)\|_{L^2} = 0$.

The techniques employed combine methods from differential geometry, including the analysis of hyperkähler metrics and associated Kähler forms $\omega_I, \omega_J, \omega_K$ satisfying the quaternionic relations, with moduli-theoretic tools and the analytical study of elliptic operators arising from the Hitchin equations. The proof relies on establishing comparison estimates between the hyperkähler distance and suitable Sobolev norms, then relating these to the L^2-norm through elliptic regularity theory.

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We illustrate our results with explicit computations on a model hyperelliptic curve with branch points at $\{0, 1, 2, 3, 4, 5, \infty\}$, computing distances and norms for various representative Higgs bundles. These examples include the zero Higgs field $(V, 0)$ which lies in the fixed-point locus with $d_{\mathrm{hyp}} = 0$, off-diagonal configurations with

$$\Phi = \begin{pmatrix} 0 & \omega_1 \\ \omega_1 & 0 \end{pmatrix}$$

yielding $\|\Delta(\Phi)\|_{L^2} = 2\sqrt{2}$, and general diagonal fields demonstrating how the distance scales with the coefficients in the expansion along the basis of holomorphic differentials.

Our work opens several directions for future investigation, including extensions to higher genus and rank, applications to mirror symmetry where Lagrangian submanifolds play a fundamental role as branes in the A-model, and connections to mathematical physics where fixed-point loci appear naturally in gauge theories with orientifold planes and supersymmetric field theories on non-orientable surfaces.

***K*eywords** Higgs bundles · Hyperelliptic involution · Lagrangian manifolds · Hyperkähler geometry · Moduli spaces

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THE TRANSPOSE-INVERSE INVOLUTION ON $\mathrm{SL}(n,\mathbb{C})$-HIGGS BUNDLE MODULI SPACES AND ELECTROMAGNETIC DUALITY

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ABSTRACT

In this research, we investigate the geometric and physical implications of outer automorphisms acting on the moduli space of $\mathrm{SL}(n,\mathbb{C})$-Higgs bundles over a compact Riemann surface of genus $g \geq 2$. The study of four-dimensional $\mathcal{N} = 2$ supersymmetric gauge theories has been influenced by the Hitchin-Donagi-Witten correspondence [2, 5], which establishes a precise correspondence between the physics of these theories compactified on a Riemann surface and the geometry of Higgs bundle moduli spaces. In this framework, electromagnetic dualities—transformations that interchange electric and magnetic charges while preserving the physical content—manifest as automorphisms of the Hitchin integrable system. While the classical S-duality of Seiberg-Witten theory [7, 8] has been extensively studied, the role of outer automorphisms of the gauge group remains largely unexplored from a geometric perspective. For the group $\mathrm{SL}(n,\mathbb{C})$, the outer automorphism group is $\mathrm{Out}(\mathrm{SL}(n,\mathbb{C})) \cong \mathbb{Z}/2\mathbb{Z}$, generated by the transpose-inverse automorphism $g \mapsto (g^t)^{-1}$, which acts on the Lie algebra $\mathfrak{sl}(n,\mathbb{C})$ by $X \mapsto -X^t$. This automorphism induces a natural involution on the moduli space $\mathcal{M}_{\mathrm{SL}(n)}$ of semistable $\mathrm{SL}(n,\mathbb{C})$-Higgs bundles, defined by $\tau(E,\varphi) = (E^*, -\varphi^t)$, where E^* denotes the dual bundle and φ^t is the transpose of the Higgs field with respect to the natural pairing.

The main result establishes that this involution τ preserves the hyperkähler structure on $\mathcal{M}_{\mathrm{SL}(n)}$ and acts in a highly structured way on the Hitchin fibration $\pi\colon \mathcal{M}_{\mathrm{SL}(n)} \to \mathcal{B}_{\mathrm{SL}(n)}$. The Hitchin fibration is a proper map whose base $\mathcal{B}_{\mathrm{SL}(n)}$ parametrizes the space of characteristic polynomials and whose generic fibers are abelian varieties encoding the electromagnetic charge lattice. We prove that τ acts on the base by reversing the signs of odd-degree invariant polynomials while preserving even-degree ones, and on the homology $H_1(A_u,\mathbb{Z})$ of a generic fiber A_u by multiplication by the sign $\varepsilon(n) = (-1)^{\lfloor (n-1)/2 \rfloor}$. This sign exhibits a striking periodicity modulo 4: it equals $+1$ when $n \equiv 0, 1 \pmod 4$ and -1 when $n \equiv 2, 3 \pmod 4$. The proof involves analyzing how the transpose operation interacts with the spectral curve construction developed by Beauville-Narasimhan-Ramanan [1] and understanding the induced map on Prym varieties following Mumford's classical theory [6].

Translating these results to physics via the Hitchin-Donagi-Witten correspondence yields a new electromagnetic duality. In the physical theory, BPS states are characterized by electromagnetic charges $\gamma \in H_1(A_u,\mathbb{Z})$, and their multiplicities $\Omega_u(\gamma)$ encode the spectrum of stable particles at a given vacuum labeled by $u \in \mathcal{B}_{\mathrm{SL}(n)}$. The geometric involution induces a physical duality relation of the form $\Omega_u(\gamma) = \Omega_{\sigma_n(u)}(\varepsilon(n)\cdot\gamma)$, where σ_n denotes the induced map on the Coulomb branch. The qualitative nature of this duality depends dramatically on the rank modulo 4. For $n = 2$, corresponding to the classic $\mathrm{SU}(2)$ Seiberg-Witten theory, the duality becomes trivial as both σ_2 and the charge transformation reduce to the identity, imposing no constraints beyond the standard S-duality. For $n = 3$ and $n = 4$, the duality manifests as a charge conjugation symmetry that relates BPS spectra at different points of the Coulomb branch by reversing electromagnetic charges,

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providing non-trivial constraints on the wall-crossing behavior studied by Gaiotto-Moore-Neitzke [3, 4]. For $n = 5, 6$ and higher ranks in the same congruence classes, the duality preserves charges while relating physically distinct vacua, imposing equality constraints on BPS multiplicities that must hold throughout wall-crossing transitions.

Thus, this work reveals that outer automorphisms provide a systematic source of new dualities whose properties depend on subtle number-theoretic patterns in the rank, and opens numerous directions for future investigation including connections to the geometric Langlands program, and applications to mirror symmetry and derived categories.

***K*eywords** Hitchin fibration · Higgs bundles · Outer automorphisms · BPS spectrum · Electromagnetic duality

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IMPROVING RECALL IN IMAGE-BASED DEEP WEB CONTENT EXTRACTION: A STATISTICAL STUDY OF WEID AND WEIDJ

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ABSTRACT

Main content extraction from Deep Web pages is challenging due to dynamic layouts and the increasing use of image-based content. This paper presents two extraction techniques, WEID and WEIDJ, designed to identify main content regions from web page images. The methods model page layouts as structured representations and apply quantitative criteria to separate informative regions from non-content elements. A statistical evaluation framework is used to assess extraction performance based on precision, recall, and F1-score. Experiments were conducted on a dataset comprising 54,321 images collected from 10 websites, covering diverse and complex layout structures. Results demonstrate that WEIDJ achieves a precision of 99% and improves recall by 85% over WEID, which attains a precision of 60% and a recall of 78%, reflecting a more balanced but less complete extraction profile. The overall average F_1-score across all experiments reaches 86%, confirming the strong effectiveness of the proposed framework. Beyond extraction performance, the proposed techniques support downstream AI and machine learning pipelines by enabling reliable content isolation for training data preparation, annotation workflows, and feature generation. The findings underscore the critical role of rigorous statistical evaluation in advancing image-based data extraction within AI-driven web mining research.

***K*eywords** Deep Web · content extraction · WEID · WEIDJ · precision and recall ·

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DATA-DRIVEN IDENTIFICATION OF GOVERNING EQUATIONS FOR SHORELINE EVOLUTION USING SATELLITE-BASED OBSERVATIONS AND SINDY

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ABSTRACT

Coastal shoreline evolution is a nonlinear and time-dependent process controlled by wave climate, sediment transport, sea-level variations, river inputs, and anthropogenic interventions. Reliable identification of these dynamics is essential for erosion assessment, sustainable coastal management, and coastal infrastructure planning. Although process-based coastal morphodynamic models provide physically meaningful descriptions of shoreline change, their applicability may be limited under complex field conditions because of simplified assumptions, parameter uncertainty, and site-specific nonlinear behaviour [1]. Recent advances in satellite-based remote sensing and open-access shoreline extraction tools have enabled long-term monitoring of coastal change and increased the potential of data-driven modelling in coastal engineering [2]. This study proposes a data-driven and physically interpretable framework for identifying the governing mathematical relationships of shoreline evolution. The Antalya–Lara coastal section in Türkiye is selected as the study area, and satellite-based shoreline observations are considered together with environmental forcing parameters. The proposed methodology is based on Sparse Identification of Nonlinear Dynamics (SINDy), which discovers compact governing equations from observational data by selecting the dominant active terms from a broad candidate function library [3]. Since shoreline evolution is influenced by external forcing mechanisms, the controlled extension of this method, SINDy-c, is also considered to incorporate variables such as wave height, river discharge, and sea-level variation into the model structure [4]. Unlike black-box machine learning approaches that mainly focus on prediction accuracy, the proposed framework aims to reveal interpretable mathematical expressions that support the physical understanding of coastal morphodynamic processes. Therefore, this study presents a transparent system identification approach that bridges conventional physics-based coastal modelling and modern data-driven equation discovery.

***K*eywords** Keywords Shoreline evolution · Coastal morphodynamics · SINDy · Equation discovery

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REVERSIBLE CANONICAL MEASURE FOR A ZERO-RANGE PROCESS WITH INDEPENDENT MARKOV NODE FAILURES

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ABSTRACT

We consider a conservative particle transport model on a network with independent Markov node failures: an edge is open only when both its endpoints are active.

Let $G_0 = (V, E)$ be a finite connected undirected graph, $|V| = N \geq 2$, with vertex degrees k_i, $i \in V$. The rates of the zero-range process (ZRP) are given by a function $u\colon \mathbb{Z}_+ \to [0, \infty)$ with $u(0) = 0$, $u(n) > 0$ for $n \geq 1$; set $p(0) := 1$, $p(m) := \prod_{n=1}^{m} u(n)^{-1}$ for $m \geq 1$. At each vertex an independent two-state Markov chain $I_v(t) \in \{0, 1\}$ is given with rates $\alpha > 0$ $(1 \to 0)$ and $\beta > 0$ $(0 \to 1)$; set $a := \beta/(\alpha + \beta)$, and let $\mu_a := \bigotimes_{v \in V} \mathrm{Bernoulli}(a)$, which is reversible since $a\alpha = (1 - a)\beta$. The random subgraph $G(t) := (V, \{\{i, j\} \in E : I_i(t) I_j(t) = 1\})$ is uniquely determined by $I(t)$. The joint Markov process $(\eta(t), I(t))$ on $\mathbb{Z}_+^V \times \{0, 1\}^V$ has generator

$$\mathcal{L}^{\mathrm{tot}} f(\eta, I) = \sum_{i \in V} \sum_{j \sim i} I_i I_j \, \frac{u(\eta_i)}{k_i} \big[f(\eta^{i \to j}, I) - f(\eta, I) \big] + \mathcal{L}_I f,$$

where $\eta^{i \to j}$ denotes the configuration after a particle has moved from i to j (the term vanishes if $\eta_i = 0$ since $u(0) = 0$); $\mathcal{L}_I$ is the generator of $|V|$ independent two-state chains.

Theorem 1. *Let $\{q_i\}_{i \in V}$ be positive weights; let $\{A_{ij}(I)\}_{i,j \in V}$ be symmetric conductances [3]: $A_{ij}(I) = A_{ji}(I) \geq 0$, $A_{ij} \equiv 0$ if $\{i, j\} \notin E$; let $\mathcal{L}_I$ be the generator of an irreducible Markov chain on $\{0, 1\}^V$ with reversible measure μ, independent of η. Then for the generator*

$$\widetilde{\mathcal{L}} f(\eta, I) = \sum_{i,j \in V} A_{ij}(I) \, \frac{u(\eta_i)}{q_i} \big[f(\eta^{i \to j}, I) - f(\eta, I) \big] + \mathcal{L}_I f$$

and any $M \in \mathbb{Z}_+$ the canonical measure

$$\pi_{N,M}(\eta, I) = \frac{1}{Z_{N,M}} \, \mu(I) \prod_{i \in V} p(\eta_i) \, q_i^{\eta_i} \cdot \mathbf{1}\left\{ \sum_{i \in V} \eta_i = M \right\}$$

is reversible with respect to $\widetilde{\mathcal{L}}$ and unique on the sector $\{\sum_i \eta_i = M\}$ provided the joint chain is irreducible. In the principal example $q_i = k_i$ (the unnormalised invariant measure of the simple random walk on G_0), $A_{ij}(I) = I_i I_j \, \mathbf{1}_{\{i \sim j\}}$ and $\mu = \mu_a$; the joint chain is irreducible whenever G_0 is connected, $N \geq 2$ and $\alpha, \beta > 0$, and the reversible measure takes the explicit form

$$\pi_{N,M}(\eta, I) \propto \prod_{i \in V} p(\eta_i) \, k_i^{\eta_i} \cdot \prod_{i \in V} a^{I_i} (1 - a)^{1 - I_i} \cdot \mathbf{1}\left\{ \sum_i \eta_i = M \right\}.$$

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Consequences for the η-marginal. (i) In the principal example the η-marginal of $\pi_{N,M}$ coincides with the classical ZRP canonical measure [1], $\pi^{\eta}_{N,M}(\eta) \propto \prod_i p(\eta_i)\, k_i^{\eta_i} \mathbf{1}\{\sum \eta_i = M\}$, for all $\alpha, \beta > 0$ and without any limiting procedure in the activation rates; in particular, for any sequence of base graphs $\{G_N\}$ on which the failure-free ZRP admits a thermodynamic limit with critical density and condensation [2], symmetric independent node activity does not shift the static phase diagram nor relocate the condensate. (ii) In the regime $\alpha + \beta \to \infty$ with a fixed, standard averaging over the fast environment yields the generator $a^2 \mathcal{L}_{\eta}^{\mathrm{ZRP}}$ for the η-projection: the dynamics is the failure-free ZRP slowed down by a factor a^{-2}. Within this reversible product-form class, a non-trivial shift of ρ_c is possible only through a modified single-site invariant measure, η-dependent failures, or a non-reversible edge dynamics with a different stationary structure.

***K*eywords** zero-range process · reversible canonical measure · Markov node failures · condensation · random environment

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ON FUZZY JACOBSTHAL NUMBERS AND THEIR ALGEBRAIC PROPERTIES

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ABSTRACT

In this paper, we investigate a novel fuzzy extension of the classical Jacobsthal number sequence, called fuzzy Jacobsthal numbers. By embedding the Jacobsthal recurrence relation into the framework of fuzzy set theory, each term of the sequence is represented as an interval-valued quantity parameterized by a fuzziness coefficient $\alpha \in [0, 1]$. This parameter regulates the uncertainty width of each term, such that the classical Jacobsthal sequence is recovered when $\alpha = 1$, whereas maximal uncertainty occurs for $\alpha = 0$. A series of new theoretical results are established. First, the well-definedness and structural consistency of the fuzzy intervals are rigorously proven. Second, the fuzzy Jacobsthal numbers are represented through matrix and interval companion matrix formulations, enabling algebraic manipulation within the framework of interval arithmetic. Third, the spectral characteristics of the associated interval matrices are analyzed to determine asymptotic behavior and stability conditions. The proposed model not only generalizes the Jacobsthal sequence into the fuzzy domain but also reveals a clear connection between fuzziness propagation and exponential growth dynamics. These results provide a systematic foundation for the development of fuzzy recurrence relations and open avenues for uncertainty quantification in discrete dynamical systems.

***K*eywords** Fuzzy Jacobsthal numbers · Interval analysis · Matrix representation · Spectral radius · Stability theory · Uncertainty propagation

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NEW APPROACH FOR SOLVING THE MATRIX DIFFERENTIAL EQUATION OF THE SECOND ORDER. APPLICATIONS

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ABSTRACT

Linear matrix differential equations of the second order play a crucial role in various fields of pure and applied mathematics, such as the calculus, the control theory and the engineering, as well as in other fields of exact and applied sciences (see, for instance, [2, 4, 6, 7], and references therein). Apostol (see [1]) and Kolodner (see [3]) have studied the following matrix differential equation of the second-order

$$X''(t) = AX(t), \tag{1}$$

submitted to the initial data $X(0)$ and $X'(0)$, where $A \in \mathbb{C}^{d\times d}$ the algebra of square matrices of order $d \times d$. Apostol and Kolodner gave some properties of the solutions of Equation (1). In [1], Apostol proposes a process for solving equation (1) based on Putzer's method to calculate the exponential of matrices (see [1, 5]). Furthermore, the second order linear matrix differential equations appear in several mechanical topics, among them the mass-spring differential systems.

This talk concerns the generalized Apostol-Kolodner differential equations of the second order. Our approach is based on some matrix square root properties, the Fibonacci-Hörner decomposition of matrix powers, and its related dynamical solution. Various explicit compact formulas for the solutions of the generalized Apostol-Kolodner matrix differential equations are established. Finally, to validate the results and show their robustness, examples and application the mass-spring differential systems are provided.

***K*eywords** Linear matrix differential equations · Apostol-Kolodner equation · Fibonacci-Hörner decomposition · Dynamical solution · Companion block matrix · Mass-spring differential systems.

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BAYESIAN AND FREQUENTIST INFERENCE FOR THE WEIBULL-ACD MODEL WITH CALENDAR EFFECTS IN HIGH-FREQUENCY FINANCIAL DATA

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ABSTRACT

This study develops an enhanced Weibull Autoregressive Conditional Duration (W-ACD-X) framework designed to endogenously incorporate calendar-effect variables for the simultaneous modeling of intraday seasonality and stochastic duration clustering. Unlike traditional ACD methodologies that rely on a two-step pre-filtering approach, which can distort underlying data dynamics and introduce statistical artifacts, the proposed W-ACD-X model employs a one-step simultaneous estimation strategy that preserves the integrity of raw high-frequency data.To ensure methodological robustness, the study employs a dual-estimation approach: Frequentist Maximum Likelihood Estimation (MLE) and a Bayesian framework based on Hamiltonian Monte Carlo (HMC) with the No-U-Turn Sampler (NUTS). The W-ACD-X specification is rigorously compared with Gompertz-ACD-X and Lomax-ACD-X models through both Monte Carlo simulations and an empirical application to high-frequency IBM transaction data. The Weibull specification consistently provides a superior fit, outperforming alternative distributions across AIC, BIC, WAIC, and LOOIC metrics. While both estimation frameworks yield consistent point estimates indicating high duration persistence, the Bayesian NUTS approach offers additional advantages by enabling comprehensive quantification of parameter uncertainty through posterior distributions. The proposed framework offers a robust and precise methodology for practitioners in operations research to model complex market microstructure without information loss associated with external data adjustment.

***K*eywords** Bayesian inference · W-ACD-X model · High-Frequency Data · Simultaneous estimation · Intraday seasonality · Weibull distribution

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STATIONARY DISTRIBUTION OF A MULTI-CLASS QUEUEING NETWORK WITH UNRELIABLE NODES ON RANDOM GRAPHS

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ABSTRACT

The classical open queueing networks of Jackson [1], BCMP [2], and Kelly [3] type admit a product-form stationary distribution under a stochastic routing matrix that does not depend on the state of individual nodes. When nodes may fail and routing adapts to the current working subnetwork, both the correct problem statement and the description of the stationary regime require a separate analysis: the effective routing matrix becomes a random function of the failure configuration, and the working subnetwork may even become disconnected. Such models arise naturally for ad-hoc wireless networks, sensor grids and transport overlays, where infrastructure is sparse, unreliable and geometrically constrained. The present work proposes such a setting in the multi-class case on a random geometric graph [6] and describes its stationary regime in closed form, in the *quenched* sense (conditionally on the failure configuration).

Let η be a homogeneous Poisson point process on $\mathbb{R}^2$ and let $G^{(R)}$ be the finite subgraph of the Delaunay triangulation $\mathrm{Del}(\eta)$ induced by a window $W \subset \mathbb{R}^2$. Each node $v \in V_R$ carries an independent Bernoulli flag $\xi_v \in \{0,1\}$ (working/failed). A fixed compact *service region* $\Lambda_0 \subset \mathbb{R}^2$ partitions the nodes into PS servers of rate μ inside Λ_0 and IS (infinite-server) transit nodes outside; external Poisson streams of rate γ_s arrive only at PS nodes $s \in \Lambda$. On its first service each customer is assigned a destination $w \in \Lambda$ by a tag matrix R^0, and is then routed step by step along the *current* shortest path $\sigma(s,w;\, G(\xi))$ of the working subgraph $G(\xi) := G^{(R)}[\{v : \xi_v = 1\}]$; if w becomes unreachable in $G(\xi)$, the customer is lost.

Fix $\xi \in \{0,1\}^{V_R}$ and let $\Lambda_\xi := \{u \in \Lambda : \xi_u = 1\}$, $T_\xi := \{z \in V_R \setminus \Lambda : \xi_z = 1\}$. Let P^ξ be the class-resolved routing matrix induced by tag-shortest-path routing on $G(\xi)$, let $(a_c^\xi)_c$ be the unique non-negative solution of the multi-class traffic equations for P^ξ, and define the load

$$\rho_u^\xi \;:=\; \frac{1}{\mu} \sum_{\substack{c \in \mathcal{K}_\xi: \\ \mathrm{loc}(c)=u}} a_c^\xi, \qquad u \in \Lambda_\xi.$$

Theorem 1. *Assume $\rho_u^\xi < 1$ for every $u \in \Lambda_\xi$. Then the reduced multi-class process on $\Lambda_\xi \cup T_\xi$ is positive recurrent, and its unique stationary distribution has Kelly-type product form: $N_u \sim \mathrm{Geom}(\rho_u^\xi)$ independent over $u \in \Lambda_\xi$; $m_{z,w} \sim \mathrm{Pois}(a^\xi_{(z,w)}/\mu)$ independent over $z \in T_\xi$; conditionally on N_u, the class composition $(n_{u,c})_c$ is multinomial with weights proportional to $a^\xi_{(u,c)}$. Moreover, the detour load on $u \in \Lambda_\xi$ admits the* exact *representation*

$$\rho^\xi_{u,\mathrm{tag}} \;=\; \frac{1}{\mu} \sum_{s \in \Lambda_\xi} \gamma_s \sum_{\substack{w \in \Lambda_\xi \\ w \neq s}} r^0_{sw}\, \mathbf{1}\big\{u \in \sigma(s,w;\, G^{(R)}(\xi)) \setminus \{s,w\}\big\}.$$

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Remarks and special cases. (i) In the failure-free regime $\xi \equiv 1$ one has $G(\xi) = G^{(R)}$ and $\Lambda_\xi = \Lambda$; the statement reduces to a Kelly-type product form on the full Delaunay subgraph, recovering the classical multi-class setting [3,4]. (ii) In the single-class case the detour-load representation reduces to a standard accounting of pass-through traffic at $u \in \Lambda_\xi$, and the geometric marginals at PS nodes match the Jackson form [1,5]. (iii) The result is *quenched* in ξ: the stationary distribution is described as a function of the failure configuration, with explicit dependence through Λ_ξ, T_ξ and the shortest-path structure of $G(\xi)$. Averaging over ξ under the Bernoulli product measure (annealed regime), and the dependence of the stability boundary $\max_u \rho_u^\xi < 1$ on the geometry of Λ_0 and on the working-cluster percolation in $G^{(R)}$, are natural directions for further analysis.

***K*eywords** multi-class queueing networks · product-form stationary distribution · unreliable nodes · random geometric graphs · Delaunay triangulation · Kelly networks

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AN INVERSE STEFAN-TYPE PROBLEM FOR AN ANTI-PARABOLIC EQUATION WITH DOUBLE FREE BOUNDARIES

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ABSTRACT

Free boundary problems arise in many areas of fluid dynamics, where it is necessary to determine simultaneously both the solution and either a part of the unknown domain or the boundary itself. The problem becomes more complex when the process is described by anti-parabolic equations in a domain D with two unknown boundaries. In this study the following problem

$$\frac{\partial u(t,x)}{\partial t} + \frac{\partial^2 u(t,x)}{\partial x^2}, \quad (t,x) \in D, \tag{1}$$

$$u(0,x) = \varphi(x), \quad x \in [0,\ell], \quad t > 0, \tag{2}$$

$$u\left(t, \gamma_k(t)\right) = \psi_k(t), \quad t \geq 0, \quad k = 1,2, \tag{3}$$

$$\frac{\partial u(t,x)}{\partial x}\Big|_{x=\gamma_k(t)}, \quad t \geq 0, \quad k = 1,2, \tag{4}$$

is considered. Here $\varphi(x)$, $x \in [0,\ell]$, and $\psi_k(t)$, $t > 0$ are given continuous functions, while $u(t,x)$ and $\gamma_k(t)$, $t \geq 0$, $(k = 1,2)$ are the unknown functions.

The novelty of this work is the constructive reduction of the double free boundary problem to a system of second-kind Volterra integral equations for the unknowns $u(x,t)$ and $\gamma_k(t)$, $(k = 1,2)$, providing a unified analytical framework for the simultaneous determination of the solution and both moving boundaries.

Keywords Free boundary problem · Anti-parabolic equation · Inverse Stefan problem

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ON DUCCI SEQUENCES WITH LEONARDO NUMBERS AND THEIR MATRIX REPRESENTATIONS: APPLICATIONS

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ABSTRACT

Leonardo numbers form a Fibonacci-related integer sequence that has recently attracted considerable attention because of its rich algebraic structure and its links to combinatorics, coding theory, and matrix analysis. In this study, we examine the n-tuples $D_{Le} = (Le_1, Le_2, \ldots, Le_n)$ and $D_{GLe} = (GLe_1, GLe_2, \ldots, GLe_n)$, formed respectively from the Leonardo numbers Le_n and the Gaussian Leonardo numbers GLe_n, and study their behaviour under the Ducci map $D : \mathbb{C}^n \to \mathbb{C}^n$, defined by taking absolute differences of cyclically adjacent entries. We derive the first and second Ducci iterates D_{Le}, D^2_{Le} and D_{GLe}, D^2_{GLe} in closed form, and use these iterates to construct circulant matrices whose entries are Leonardo and Gaussian Leonardo numbers. Building on this construction, we design a dual-layer coding and decoding algorithm: a plaintext message matrix is first encoded by left multiplication with a Leonardo (or Gaussian Leonardo) circulant matrix generated under the Ducci map, and the resulting matrix is then perturbed by the incidence matrix of an auxiliary labelled graph, so that the overall security depends jointly on the algebraic structure of the circulant matrix and the combinatorial choice of the graph. Explicit numerical examples, including full encoding and decoding of sample text messages, illustrate the practicality of the scheme. In the second part of the study, we investigate determinantal and permanental representations associated with Leonardo numbers under the Ducci map. Using the characteristic polynomial of an associated tridiagonal matrix and its connection to Chebyshev polynomials of the second kind, we obtain a closed-form expression for the relevant determinant in terms of Fibonacci numbers. We further apply the contraction method for permanents to a Hessenberg-type matrix and establish a permanental identity involving Leonardo and Fibonacci numbers. A Maple 13 source code implementing the contraction algorithm is also provided so that the theoretical results can be verified computationally. Overall, the paper contributes new links between Ducci sequences, Leonardo-type number families, circulant and Hessenberg matrices, and offers a graph-based cryptographic application of independent interest.

Keywords Leonardo numbers · Gaussian Leonardo numbers · Ducci map · circulant matrices · coding theory

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A GENERAL IMPLICIT THEOREM AND VARIOUS TYPES OF $F-$CONTRACTIONS IN GENERALIZED $G-$METRIC SPACES

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ABSTRACT

The further development of the fixed point theory and contractive mappings is still interesting and active topic for research associated with applications in pure and applied mathematics. In this presentation talk, our aim is to give some new fixed point theorems for types of $F-$contractions and $(\alpha, F)-$ contractions, by investigating the existence and uniqueness of fixed point for a self-mapping defined on a generalized $G-$metric space. By using this implicite approach the results unifies and extend the recent works in this domain.

Keywords $G-$ metric space $\cdot$ $F-$contraction $\cdot$ Fixed Point

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COATOMS AND THE BOOLEAN IMAGE IN THE $\mathcal{R}_1$-LATTICE ON T_{01} FOR 3-VALUED LOGIC

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ABSTRACT

Let $E_3 = \{0,1,2\}$ and let P_3 denote the clone of all operations on E_3. By Post [1] the lattice of clones on a two-element domain is countable; by Yanov and Muchnik [2] it already has the cardinality of the continuum on any finite domain of cardinality at least three, ruling out a Post-style enumeration on E_3. A standard response, the *strengthened-closure programme*, is to enrich the clone closure by an additional invariance whose lattice of closed sets is more tractable [3,5].

We single out the value $2 \in E_3$ as *distinguished* and consider the equivalence $f \equiv_{01} g$ iff $\mathrm{ar}(f) = \mathrm{ar}(g)$ and $f^{-1}(2) = g^{-1}(2)$. The operator $\mathcal{R}_1$ is the smallest closure operator on P_3 that contains the projections, is closed under superposition and the standard arity equivalence, and additionally identifies $\equiv_{01}$-equivalent functions. In the language of matrix logics in the sense of Wójcicki [4], $\equiv_{01}$ is precisely the relation of $\mathfrak{M}$-indistinguishability for the three-valued matrix $\mathfrak{M} = (E_3, P_3, \{0,1\})$: $\mathcal{R}_1$ records the input set producing the non-designated value while collapsing all internal differences within the designated set.

For the natural ambient class $T_{01} := \{f \in P_3 : f(\{0,1\}^n) \subseteq \{0,1\}\}$ — operations whose restriction to the Boolean cube is itself Boolean, see [5] — we obtain a complete classification of maximal proper $\mathcal{R}_1$-closed subclasses.

Theorem 1. *The class T_{01} has exactly three $\mathcal{R}_1$-precomplete subclasses, namely*

$$T_{01} \cap T_2, \qquad T_{01} \cap T_\sim, \qquad T_{01} \cap I_2.$$

Moreover, their pairwise intersections, together with the triple intersection and T_{01} itself, give at least eight pairwise distinct $\mathcal{R}_1$-closed subclasses, forming a sublattice in the lattice of $\mathcal{R}_1$-closed subclasses of T_{01}.

The $\sim$-preserving class $T_\sim$ admits a sharper description. Let $\pi\colon E_3 \to E_2$ be the projection $\pi(0) = \pi(1) = 0$, $\pi(2) = 1$, extended coordinatewise. For $f \in T_\sim$, $\sim$-preservation forces a unique $\widetilde{f} \in P_2$ with $\widetilde{f} \circ \pi = \pi \circ f$, the *Boolean image* of f.

Theorem 2. *The map $[f]_{\equiv_{01}} \mapsto \widetilde{f}$ is a clone isomorphism between the quotient $(T_\sim, \mathcal{R}_1)/\equiv_{01}$ and the standard Boolean clone (P_2, C).*

Corollary 1. *The lattice of $\mathcal{R}_1$-closed subclasses of $T_\sim$ is isomorphic to Post's lattice [1], and is therefore countably infinite.*

The two precomplete subclasses $T_{01} \cap T_2$ and $T_{01} \cap I_2$ contrast sharply with $T_\sim$: their $\mathcal{R}_1$-closed subclasses encode genuinely three-valued combinatorics through the structure of $f^{-1}(2) \subseteq E_3^n$, and need not be countable. The main theorem can therefore be read as identifying $T_\sim$ as the unique "Boolean" coatom in the $\mathcal{R}_1$-lattice on T_{01}. Symmetric variants of the classification for the value-layers T_{02} and T_{12}, obtained by relabelling the distinguished value, are also recorded; building on

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the first author's earlier work [6] on lattice structure of 01-functions, this completes a triad of layer-wise classifications under $\mathcal{R}_1$.

***K*eywords** three-valued logic · matrix logics · designated values · clone theory · closure operators · precomplete classes

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THE STABILITY OF CUBIC COMPLEX GINZBURG-LANDAU EQUATION DRIVEN WITH DRIFT AND DEFECTS

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ABSTRACT

This study investigates the complex dynamics of dissipative solitons within the cubic complex Ginzburg-Landau equation (CCGLE) under the influence of drift and defect mechanisms. Utilizing a modified variational formulation paired with pulsating soliton trial functions, we analyze the behavior and stability of the resulting Euler-Lagrange equations. Standard stability analyses via the Routh-Hurwitz criterion and Hopf bifurcation theory reveal that the system's eigenvalues fail to satisfy stability conditions, rendering the localized solutions inherently unstable. Interestingly, treating these Euler-Lagrange equations as fixed points uncovers a distinct class of non-stationary phenomena which is exploding solitons. We demonstrate that eigenvalues possessing positive real parts directly drive the explosive phases of these solutions. Our results indicate that systems with infinite degrees of freedom allow external drift and defect mechanisms to self-organize, forcing the system to oscillate periodically into explosive regimes before experiencing complete decay. Ultimately, these insights underscore the profound complexity introduced by drift and defects in the CCGLE. The observed "explosions" are characterized by high-amplitude, intermittent excursions in phase space, followed by a rapid collapse and subsequent reorganization toward localized attractors. These dynamics represent a robust self-organization mechanism inherent to driven-dissipative nonlinear media.

Keywords Dissipative solitons · Complex Ginzburg-Landau equation · Dynamical systems · Hopf bifurcation · Exploding solitons

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MATHEMATICAL MODELLING AND SUSTAINABILITY: AN INTERDISCIPLINARY INTERVENTION FOR THE TRAINING OF FUTURE ENGINEERS

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ABSTRACT

This paper analyzes an interdisciplinary educational intervention based on problem-based learning (PBL) and integrated science, technology, engineering, and mathematics (STEM) methodology [1]. Implemented with engineering students at the Coimbra Institute of Engineering, the project evaluated the impact of addressing real-world territorial challenges on students' technical competence and intrinsic motivation [2, 3]. Working as energy consultants, students designed an integrated photovoltaic solar system for the Cunha Baixa region in Mangualde, Portugal.

The pedagogical workflow followed a quantitative, single-group, pre-experimental design (pre- and post-test) spanning five phases [2, 4]. Students engaged in peer-teaching activities by producing didactic audiovisual materials for secondary education, performed geospatial optimization, applied trigonometry for seasonal solar tilt tracking, and executed fundamental numerical calculation methods. Utilizing dynamic software tools like GeoGebra and Excel, they implemented optimization routines, Newton's fourth-order interpolation to predict climate production fluctuations, and the Bisection Method to establish the self-sufficiency break-even point against local urban demands.

Inferential statistical analysis ($n = 34$, $\alpha = .84$) confirmed a highly significant enhancement in students' technical skills and perceived self-efficacy ($t(33) = 11.56, p < .001$) with an exceptionally large effect size (Cohen's $d = 1.98$). The findings demonstrate that embedding abstract mathematical calculations, such as interpolation and numerical root-finding, within practical sustainability scenarios mitigates math anxiety, enhances conceptual assimilation, and builds professional social responsibility aligned with the UN 2030 Agenda.

***K*eywords** STEM education · Mathematical modelling · Sustainable engineering · Problem-based learning · Numerical methods.

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STEAM-BASED ETHNOMATHEMATICS FOR LINEAR ALGEBRA LEARNING: A CULTURAL HERITAGE APPROACH USING DIGITAL BREAD STAMPS IN ENGINEERING EDUCATION

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ABSTRACT

This paper examines the integration of cultural heritage and digital technology in Higher Education through an innovative STEAM-based project-based learning (PBL) intervention [1, 2]. Implemented within a first-year Linear Algebra course for Biomedical and Electromechanical Engineering students, the project titled "Marking the Bread in the Digital Communal Oven" addresses the abstract nature of vector spaces and matrix operations [3]. It uses traditional Portuguese bread stamps (pintadeiras or cunhos), historically documented in ethnographical collections, as a conceptual framework for teaching matrix transformations [4]. The pedagogical methodology followed a three-phase design: ethnographic analysis, mathematical modelling, and computational implementation [2, 4]. Using computational tools such as MATLAB, GeoGebra, and Generative AI, students translated the physical geometric patterns of historical bread stamps into digital matrices. This approach allowed them to visualize and execute geometric transformations, such as reflections, rotations, and dilations, by directly manipulating matrix equations. Quantitative and qualitative analysis ($n = 51$) demonstrated high student academic achievement and engagement, with a significant majority developing positive perceptions toward mathematics [3]. By connecting ethnomathematics with modern computational software, the intervention successfully transformed abstract algebraic structures into meaningful, tangible engineering applications [1, 2]. The study concludes that embedding local cultural heritage within STEM curricula enhances students' spatial reasoning, improves conceptual understanding of linear transformations, and fosters a sense of regional identity and social responsibility among future engineers [4].

***K*eywords** Linear Algebra · STEAM Education · Cultural Heritage · Matrix Transformations · Generative AI · Ethnomathematics · Higher Education · Project-Based Learning · Engineering Education.

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A METHODOLOGY FOR TEACHING HEAT TRANSFER USING THE FINITE DIFFERENCE METHOD AND OCTAVE

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ABSTRACT

This study presents a computational methodology integrating the Finite Difference Method (FDM) and Octave to teach heat conduction in solids. The articulation between mathematical theory, physical modeling, and computational practice constitutes a pedagogical strategy that can promote a deeper understanding of thermal phenomena and the development of skills in scientific programming, computational thinking and numerical thinking.

FDM approximates derivatives through finite-difference operators derived from Taylor series expansions, transforming differential equations into discrete algebraic systems that describe temperature and heat flow. Spatial and temporal discretization allows systematic analysis of numerical stability, convergence, and numerical errors. Such analysis is essential for ensuring that the discrete solution reliably represents the underlying physical behavior, particularly in transient or highly non-linear thermal processes. Octave, an open-source *software* with Matlab-like syntax, significantly expands the possibilities for numerical experimentation. Its flexibility allows the use of different types of meshes, parameter variation (e.g., thermal diffusivity, step sizes), and graphical visualization of results.

The integrated approach emphasizes teacher mediation, guiding students through the physical interpretation of the problem, the formulation of appropriate mathematical modeling, and computational implementation. This mediation is essential to ensure the meaningful use of technology, avoiding the mechanical execution of code without understanding the underlying physical principles.

This study follows a quasi-experimental research design, conducted in the 2023/2024 academic year with 17 fourth-year students of the Bachelor's Degree in Mathematics and Physics Teaching at the Higher Pedagogical School of Bié (Angola). The data, of a mixed nature through questionnaires, pre- and post-tests, and analyzed using content analysis and statistical techniques were collected. Academic performance was assessed using Hake's normalized gain equation revealing a 70 % average improvement in student performance.

Teacher mediation ensured meaningful technology use, promoting skills in computational modeling, critical thinking, and interdisciplinary integration.

Integrating FDM with Octave in heat transfer studies links theory to practice, reinforcing essential mathematical skills such as discretization, numerical stability analysis, and interpretation of differential equations. This approach prepares students for the challenges of thermal engineering, and computational modeling, promoting active and meaningful leaning supported by simulation and experimentation. It also allows students to relate mathematical reasoning to physical interpretation and computational implementation. As a scalable methodology, it offers an alternative to traditional lectures, aligned with contemporary active learning methodologies.

***K*eywords** Integrated Approach. Finite Difference Method. Octave *Software.* Heat Transfer Problems.

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ELABORATION OF A NEW CENTERED GAUSSIAN PROCESS

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ABSTRACT

In this work, firstly, we introduce a new gaussian process as an extension of the well known bifractional Brownian motion as a linear combination of a finite number of independent bifractional Brownian motions. We have chosen to call this process the mixed bifractional Brownian motion. Secondly, we study some stochastic properties and characteristics of this process: The Holder continuity, the self similarity, the quadratic variation, the Markov property and the differentiability of the trajectories, the long-range dependence, the stationarity of the increments and the behavior of the noise generated by the increments of this process. We believe that our process can be a possible candidate for models which involve self similarity, long range dependence and non-stationarity of increments.

***K*eywords** Gaussian process · Brownian motion · Quadratic variation

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DISTRIBUTED FILTERING FOR NETWORKED SYSTEMS UNDER STOCHASTIC UNCERTAINTIES AND FDI ATTACKS

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ABSTRACT

Signal estimation in multi-sensor stochastic systems has grown into a significant research area due to its high relevance in numerous applied and theoretical fields. Within this domain, distributed estimation has attracted considerable attention. This approach is particularly useful when sensor nodes are spatially distributed according to a predetermined network topology, allowing each node to act as a local fusion center. By combining its own measurement data with information received from neighboring nodes, each sensor can significantly improve estimation performance. This collaborative strategy offers distinct advantages, including ease of implementation, robustness, scalability, and high reliability. While the foundations and challenges of these multi-sensor architectures have been extensively surveyed in recent literature (e.g., [1], [2], [3]), networked systems remain inherently prone to simultaneous random flaws and cyber-threats which, if not appropriately modeled, can substantially deteriorate estimator performance.

To address these challenges, this paper investigates the distributed linear filtering problem for systems where sensor measurements are simultaneously affected by random parameter matrices ([4], [5], [6]), perturbed by time-correlated additive noises ([4], [7]), and vulnerable to linear false data injection (FDI) attacks ([8], [9]). The inclusion of random parameter matrices provides a unified framework that covers a wide range of common network-induced uncertainties, such as multiplicative noise, sensor gain degradation, and missing measurements. Furthermore, the infinite-step time correlation of the measurement noises makes the algorithm highly suitable for high-frequency sampling applications where noise is significantly correlated over consecutive periods. Additionally, the stochastic nature of the FDI attacks is modeled via time-varying Bernoulli random variables, which reflect realistic cyber-security vulnerabilities where different nodes experience distinct random attacks.

Under a covariance-based methodology, the proposed distributed filtering algorithm operates in two stages: first, intermediate estimators based on local and adjacent node measurements are designed; second, these intermediate estimators from neighboring sensors are combined using least-squares matrix-weighted linear combinations. Notably, this covariance-driven technique does not require prior knowledge of the signal evolution model and provides optimal estimators under the mean squared error criterion without additional structural assumptions.

Keywords Distributed estimation · Networked stochastic systems · Random parameter matrices · Time-correlated noise · False data injection attacks · Covariance-based filtering

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ON DETERMINANT REPRESENTATIONS AND GENERATING FUNCTIONS FOR GENERALIZED AVICENNA NUMBERS

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ABSTRACT

In this talk, we revisit the generating function for the recently introduced generalized Avicenna numbers. Using the Hessenberg determinant representation for linear recurrence relations, we show that the rational generating function of this family of sequences follows from a general structural framework. We also consider a class of sequences defined by binomial-coefficient recurrences, which we refer to as Pascal row recurrences, and derive their generating functions in closed form. These generating functions have denominator $(1-t)^r$, with an explicit numerator determined by transformed initial conditions.

***K*eywords** Avicenna numbers · Generating functions · determinant · Pascal triangle · Hessenberg matrix

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First Synthesis of Recent Research on 21stCentury Skills in Higher Education

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Abstract

The rapid evolution of professional, technological, and societal demands has positioned 21st century skills (SKILLS21) as essential learning outcomes in higher education. This systematic review synthesizes evidence from seventeen studies published between 2021 and 2026 to examine how these skills are conceptualized, developed, and assessed within contemporary higher-education contexts, with particular emphasis on Science, Technology, Engineering, and Mathematics (STEM) and engineering disciplines. Using a thematic synthesis approach, the review identifies strong convergence around core competencies, including collaboration, communication, creativity, critical thinking, metacognition, leadership, digital literacy, and interdisciplinary or boundary crossing abilities. The findings demonstrate that active and authentic pedagogies—such as project based learning, client sponsored work, interdisciplinary teamwork, work based learning, and technology enhanced STEM activities—are most effective in fostering these competencies when supported by intentional scaffolding and reflective practice. Assessment practices, however, remain fragmented; while rubrics and peer evaluation tools show promise, many studies rely heavily on self report measures, underscoring the need for more robust, performance based assessment frameworks. Contextual factors, including institutional resources, industry partnerships, and co curricular opportunities, significantly shape the extent to which SKILLS21 can be meaningfully integrated into curricula. Across the examined studies, authors agree that skills such as collaboration, communication, creativity, critical thinking, metacognition, leadership, digital literacy, and boundary-crossing abilities are essential for preparing graduates to navigate complex and unpredictable environments. These skills are no longer limited to disciplinary learning; they are important for fostering students' capacity to apply knowledge, solve real-world problems, and adapt to emerging challenges. Evidences indicate that active, authentic, and socially situated pedagogies are particularly effective in fostering these skills. Still, the review also reveals that the successful development of SKILLS21 depends on intentional design, structured scaffolding, and supportive institutional conditions. Without these elements, even well-intentioned pedagogical innovations may fail to produce meaningful improvements in skills. Therefore, higher education must adopt coordinated, evidence informed strategies that align curriculum design, pedagogy, assessment, and policy to ensure that graduates are equipped with the adaptable, transferable skills required to thrive in an increasingly complex world.

***K*eywords** Higher education · 21st century skills · Educational practices · Literature review

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THE INVISIBLE STRUCTURE OF DATA: A GEOMETRIC-ALGEBRAIC READING OF MODE

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ABSTRACT

The connections between the several disciplines of Mathematics - e.g., Algebra, Geometry, Calculus, Statistics, and Arithmetic - are essential because they transform sometimes fragmented knowledge into structured, applicable knowledge, overcoming the view that mathematics is merely a set of more or less isolated contents. This is valued in various mathematics curricula across different educational levels. The connections among mathematical knowledge across numerous areas deepen understanding, stimulate logical reasoning, and promote the practical application of mathematics to solve complex real-world problems. These connections allow students to apply, e.g., numerical relationships to shapes, link patterns to algebra, and use data analysis to interpret social or scientific situations. On the other hand, visualization is a vital, proven-effective tool in mathematics education. Visualization transforms abstract concepts into concrete, understandable, and assimilable images. So, mathematics visualization promotes a deeper conceptual understanding, improves problem-solving skills, and increases student engagement, allowing them to see patterns, make connections, and develop mathematical intuition. Additionally, it has a significant positive impact on mathematics learning outcomes. By using diagramming, manipulatives, and digital tools, teachers can make mathematics an active and creative experience, thereby improving students' performance and promoting a deeper, more intuitive understanding of the subject. Within this framework, this work aims to connect Statistics, Geometry, and Algebra and to promote the visualization, through technological tools, of the knowledge involved in this connection among these mathematical areas. In particular, the objective is to establish a relationship between the statistical concept of mode (continuous data case) and the geometric figure of the parabola, mediated by algebra, including the possibility of solving systems using matrices, and complemented by the use of technology - dynamic geometry, both for visualizing the aforementioned relationship and some processes for obtaining associated formulas. In this way, the statistical concept of the mode (in the continuous case) and two distinct geometric methods for estimating this parameter, based on proportionality relationships between similar triangles, are presented, using dynamic geometry to illustrate the model-building process and its differences. Based on one of the processes for estimating the mode in those data grouped by classes, this presentation demonstrates that the abscissa of the parabola's vertex equals the mode. This equality is verified geometrically using dynamic geometry and algebraically using systems of equations. The process and the relationship are visualized, formulas and their origins are confirmed, and a range of mathematical knowledge from several levels and areas of mathematics is connected. This presentation can serve as a starting point for teachers to engage students in a deeper understanding of intuitive, interconnected mathematics. Of course, it will also stimulate logical reasoning and awareness of the practical applicability of mathematics in solving more complex and/or abstract problems, and, through a visual basis, to make mathematics an active and creative experience, aiming to improve student performance.

Keywords Statistics · Geometry · Algebra · Mode · Algebraic linear systems · Mathematical connections · Visualization · Dynamic geometry

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THE SPECTRAL STRUCTURE OF ROOTS OF $\mathcal{M}$–STURM-LIOUVILLE PROBLEMS

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ABSTRACT

In this study, classical Sturm–Liouville problems are investigated within the framework of fractional calculus by employing the $\mathcal{M}$-derivative. To establish a mathematically rigorous and physically meaningful basis for the analysis, the one-dimensional Schrödinger equation is formulated as an $\mathcal{M}$–Sturm–Liouville problem. The primary objective of the study is to examine the eigenvalue–eigenfunction relationships arising in $\mathcal{M}$–Sturm–Liouville problems, with particular emphasis on the distribution of the zeros of the corresponding eigenfunctions.

One of the main contributions of this work is the interpretation of the zeros of the eigenfunctions as a nested network structure. Furthermore, two fundamental results of classical Sturm–Liouville theory, namely the Sturm Oscillation Theorem and the Sturm Comparison Theorem, are extended to the $\mathcal{M}$-derivative framework. These generalizations provide a theoretical foundation for the investigation of the oscillatory behavior of solutions and for the comparison of eigenfunctions associated with different $\mathcal{M}$–Sturm–Liouville problems.

***K*eywords** Fractional · $\mathcal{M}$-Sturm–Liouville problem · Oscillation theorem

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FRACTIONAL SOME MATHEMATICAL MODELING IN ENGINEERING

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ABSTRACT

This study investigates beam mechanics from the perspective of fractional calculus to enhance the structural assessment of load-bearing elements. A comprehensive fractional mathematical model is proposed, incorporating the coupled effects of elastic modulus, sectional moment of inertia, external loads, and axial compressive force on beam deflection. The framework is evaluated using three distinct derivative definitions: Proportional derivative, $\mathcal{M}$-derivative, and Generalized $\mathcal{M}$-derivative. Under the proportional derivative approach, the governing second-order equation is analytically solved via the method of proportional variation of parameters based on the beam's boundary conditions. For the $\mathcal{M}$-derivative and generalized $\mathcal{M}$-derivative formulations, Laplace transforms and convolution properties are utilized. The mathematical results are validated visually through graphical analyses, providing highly efficient and accurate assessments for engineering structures.

Keywords Fractional · Mathematical modeling · Beam deflection

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THE SINGULAR SPECTRAL PROBLEM ASSOCIATED WITH GENERALIZED M-DERIVATIVE

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ABSTRACT

In this paper, we consider spectral problem with generalized $\mathcal{M}-$derivative for having Bessel potential function type. Bessel function type problems are significant in the fields of physics. The purpose of this study is to prove a more general version of the representation of the solution to the singular spectral problem using the Bessel potential function type, by including the $\mathcal{M}-$series. Additionally, the containment of the $\mathcal{M}-$series in the definition of the generalized $\mathcal{M}-$derivative enables for more elaborate conclusions to be reached with the acquired data. We employ the Laplace transform to get the representation of the solution to this singular spectral problem. The Laplace transform, in terms of its generalized $\mathcal{M}-$derivative, is a vigorous method for depth analysis of complex problems. In this singular spectral problem, the substantial results are represented with graphs through the MATLAB program [1-10].

Keywords Singular spectral problem · Generalized $\mathcal{M}-$derivative · Laplace transform · $\mathcal{M}-$series

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GENERALIZED M-SPECTRAL PROBLEM HAVING HYDROGEN ATOM POTENTIAL

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ABSTRACT

In this paper, us address generalized $\mathcal{M}-$Sturm-Liouville problem with hydrogen atom potential associated with initial conditions:

$$-D_{\mathcal{M}}^{\alpha,\beta,\gamma}D_{\mathcal{M}}^{\alpha,\beta,\gamma}y(x)+\left[\frac{l(l+1)}{x^2}-\frac{2}{x}+q(x)\right]y(x)=\lambda y(x). \tag{1}$$

In this study, our aim we obtain representation of solution for generalized $\mathcal{M}-$Sturm-Liouville problem with hydrogen atom potential by taking advantage of Laplace transform by means of generalized $\mathcal{M}-$derivative.
Additionally, us define Laplace transform by means of generalized $\mathcal{M}-$derivative as follows:

$$\mathcal{L}_{\alpha,\beta,\gamma}\{f(t)\}(s)=\left[\frac{c_1...c_r}{a_1...a_p}\frac{\Gamma(\beta+\gamma)}{\Gamma(\gamma)}\right]\int_0^\infty e^{-s\frac{\left[\frac{c_1...c_r}{a_1...a_p}\frac{\Gamma(\beta+\gamma)}{\Gamma(\gamma)}\right]t^\alpha}{\alpha}}f(t)t^{\alpha-1}dt. \tag{2}$$

This problem, important results are demonstrated thanks to graphs of use MATLAB program [1-6].

Keywords $\mathcal{M}-$series · Laplace transform · Hydrogen atom potential · Generalized $\mathcal{M}-$derivative

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THE EXPLORATORY SPATIAL DATA ANALYSIS (ESDA) OF HOUSEHOLD INCOME DISPARITIES IN MALAYSIA

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ABSTRACT

Income inequality remains a critical concern in developing countries, such as Malaysia, where rapid urbanization and regional economic disparities are prevalent. The ESDA offers a powerful framework for exploring spatial trends, identifying clusters, and assessing the relationship between income levels and geographical locations. Perhaps, this study offers the K-Means clustering analysis to give another view of clustering techniques. These methods include choropleth mapping, spatial autocorrelation analysis, and the Local Indicators of Spatial Association (LISA) to examine the spatial autocorrelation of income patterns. Our findings reveal significant spatial disparities in income distribution, with certain regions demonstrating higher concentrations of affluence, whereas others exhibit pockets of economic marginalization. By employing Global Moran's I and Local Moran's I, this study determines the presence of spatial autocorrelation and identify statistically significant clusters of income levels, shedding light on spatial trends and relationships. Moreover, LISA analysis uncovered localized hotspots and cold spots, providing valuable insights into spatial income inequalities and potential drivers of income disparities. Understanding where income disparities exist helps government officials, urban planners, and social organizations design targeted strategies for poverty reduction and regional development.

***K*eywords** Exploratory Spatial Data Analysis · Indicators of Spatial Association · Global Moran's I · Local Moran's I

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ON LEONARDO VECTORS

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ABSTRACT

In [3], the author introduced the vector representations of Fibonacci numbers, establishing several properties by using results in linear algebra, extending this framework to two-dimensional spaces. Fibonacci vectors were also explored in [1]. Subsequent the Horadam vectors was introduced their fundamental algebraic properties are derived, [1]. Recently, in [4], the authors introduced and studied the Pell and Pell–Lucas vectors, given the vectorial properties of these generalized vectors, as the cosines of the angles between these vectors.

In this work, we introduce the $n-$th Leonardo vector of lenght m, that is the vector whose the components are the nth throught $(n+m-1)-$st Leonardo numbers. From linear algebraic perspective, we study them in 2 and 3 dimensions, and define the vector product and the inner product. Moreover, we derive the recurrence relation and the matrix representation, establishing properties and identities for these new class of vectors.

***K*eywords** Fibonacci vectors · Leonardo numbers · Linear Algebra

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ENHANCING HEAT TRANSFER TEACHING THROUGH THE THEORY OF DIDACTIC SITUATIONS AND FINITE DIFFERENCE METHOD: A CASE STUDY

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ABSTRACT

The Theory of Didactic Situations (TDS) was applied to teach heat transfer using the Finite Difference Method (FDM), with the aim of fostering active, meaningful learning and developing mathematical, physical, and computational skills among undergraduate students. This quasi-experimental case study, conducted with 19 pre-service Physics and Mathematics teachers at the Higher Pedagogical School of Bié (Angola) during the 2024/2025 academic year, investigated how TDS-guided teacher mediation enhances students' understanding of FDM in modeling steady-state and transient heat conduction.

TDS structures learning through four phases: action (problem exploration and strategy formulation), formulation (development of explanations, models, and representations), validation (testing ideas, identifying errors, and refining procedures), and institutionalization (formalizing knowledge and linking it to scientific principles). In the context of heat transfer, these phases were adapted to FDM, enabling students to understand the physics of conduction while also grasping the numerical logic behind the method. As an iterative approach, FDM requires students to discretize space and time, implement algorithms, and interpret physical results, thereby fostering computational thinking (abstraction, decomposition, pattern recognition, and algorithmic reasoning).

Data were collected through questionnaires and pre- and post-tests, and analyzed using content analysis and Hake's normalized gain equation. The results revealed an average improvement of over 70% in academic performance, with significant gains in conceptual understanding of thermal conduction, procedural skills in applying FDM, active participation in the TDS phases, long-term retention through the integration of theory and practice, and the development of computational thinking as a cross-disciplinary skill. Teacher mediation ensured the meaningful use of FDM, promoting skills in modeling, critical and creative thinking, computational thinking, and interdisciplinary integration.

This study demonstrates that integrating TDS and FDM constitutes an effective didactic strategy for transforming heat transfer education. By bridging theoretical and practical knowledge, this approach strengthens the training of future Physics and Mathematics teachers, promoting deeper, more engaged, and longer-lasting learning.

***K*eywords** Theory of Didactic Situations. Finite Difference Method. Teaching Heat Transfer. Teacher Mediation.

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NONLOCAL FISHER-KPP EQUATION WITH FREE BOUNDARIES: NUMERICAL STRATEGIES FOR ACCELERATED SPREADING

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ABSTRACT

The nonlocal Fisher-KPP equation with free boundaries models population dynamics in which individuals undergo long-range dispersal across a habitat whose edges evolve in response to population flux. Solving this problem numerically is non-trivial: the coupling of a nonlocal integral operator with an explicitly moving domain precludes standard parabolic solvers and demands specialized discretization strategies. We propose and compare two numerical strategies. The Front-Tracking Runge-Kutta (FTRK) method operates directly on the physical domain, coupling fourth-order Runge-Kutta time integration with dynamic grid adaptation to preserve spatial resolution during significant domain expansion. The Front-Fixing (FF) method applies a Landau-type coordinate transformation that maps the evolving domain onto a fixed computational interval, converting the governing equation into a partial integro-differential equation (PIDE) with state-dependent advection terms; these are discretized using upwind and Lax-Wendroff schemes paired with Simpson's quadrature for the integral operator. While FTRK accommodates domain growth naturally at the cost of a progressively larger system, FF maintains a fixed number of degrees of freedom at the price of decreasing effective spatial resolution in regimes of strong expansion. Both methods are validated against theoretical criteria for the spreading-vanishing dichotomy and correctly reproduce accelerated propagation driven by fat-tailed dispersal kernels.

***K*eywords** Nonlocal diffusion · Free boundary problem · Front-tracking method · Front-fixing method · Accelerated spreading

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QUDIT QUANTUM COIN FLIPPING AND BINET FORMULAS FOR GENERALIZED FIBONACCI NUMBERS

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ABSTRACT

The quantum coin flipping problem for the generic qudit coin, with arbitrary number d of maximally random states is studied. For a given d we construct the set of orthonormal quantum coins, generated by the Hadamard gate and determined by the roots of unity. The states are the eigenstates of the number operator $N = \sum_k^{d-1} k|k\rangle\langle k|$ and of the Sylvester clock operator $\Sigma_3 = q^{2N}$, $q^{2N}|k\rangle = q^{2n}|k\rangle$, acting on the coin states as the shift operator. The clock and shift operators are the q-commutative operators, generating the quantum group symmetry for the qudit coin states. For flipping quantum coin the probability of duplicated states in a fixed number of trials is represented by generalized Fibonacci numbers. For these numbers the Binet formulas are derived. It is shown that they represented by two parametric pq-quantum calculus numbers and in particular cases, they include several sequences of numbers such as, the Mersenne numbers, the Fibonacci-Lucas numbers, and the Pell numbers. In particular, the probability for duplicated states of $d = 5$ ququit quantum coin,

$$P_n = \frac{1}{4}\left(\frac{2}{5}\right)^n \mathcal{P}_{n-1},$$

is determined by numbers

$$\mathcal{E}_n = 2^{n-1}\frac{(1+\sqrt{2})^n - (1-\sqrt{2})^n}{2\sqrt{2}} = 2^{n-1}\mathcal{P}_n,$$

where $\mathcal{P}_n = 0, 1, 2, 5, 12, 29, 70, ...$ are the Pell numbers. In case of quhexit coin with six basis states we have probability of duplicated states, determined by the Binet formula with the golden ratio $\varphi = (1+\sqrt{2})/2$ and $\varphi' = -\varphi^{-1}$,

$$H_n = 5^{\frac{n-1}{2}}\frac{\varphi^{2n} - (-1)^n\varphi'^{2n}}{3}.$$

For the even and odd index n, this formula can be rewritten in terms of Fibonacci numbers F_n and Lucas numbers L_n,

$$\begin{aligned} H_{2k} &= \frac{5^k}{3}F_{4k}, \\ H_{2k+1} &= \frac{5^k}{3}L_{4k+2}. \end{aligned}$$

Finally, we show that every n-qubit state $|\psi_k\rangle$ represents a qudit quantum coin with number of states $d = 2^n$ and the number of such coins is equal 2^n. In this case, the Binet formula is

$$D_N = \left(\frac{\sqrt{M_n}}{2}\right)^N \frac{(\sqrt{M_n} + \sqrt{M_n+4})^N - (\sqrt{M_n} - \sqrt{M_n+4})^N}{\sqrt{M_n(M_n+4)}}$$

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where

$$M_n = [n]_2 = \frac{2^n - 1}{2 - 1} = 2^n - 1,$$

are the Mersenne numbers.

***K*eywords** Quantum coin · Binet formula · Fibonacci numbers · qudit states

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THE RIEMANN ZETA FUNCTION AND QUANTUM CALCULUS OF PRIMES

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ABSTRACT

In this paper we propose some applications of quantum q-calculus to several problems related with the Riemann zeta function. For removing the pole singularity for this function at $s = 1$, we first apply the Möbius transformation, moving it to infinity. Then, we formulate and solve the q-differential equation

$$xD_q f(x) = \left[\frac{1}{2}\right]_q f(x).$$

for nontrivial zeros of zeta function. The general solution of this equation takes the form

$$f(x) = \sqrt{x}\, G_q(x),$$

where $G_q(x)$ is the q-periodic modulation function, satisfying

$$D_q G_q(x) = 0$$

and represented in the general form as

$$G_q(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\beta \ln x}.$$

This suggests that the quantum Hamiltonian operator for solution of the Riemann problem could be based on the q-deformed version of the Berry-Keating Hamiltonian operator in the form

$$H = xD_q + D_q x.$$

Finally, we derive the Planck type formula for the gas of prime numbers in terms of q-numbers,

$$\langle E \rangle_N = \hbar\omega q \frac{d}{dq} \ln[N+1]_q$$

representing the truncated Riemann zeta function as

$$\zeta_N(s) = \frac{1 - p_1^{-s(N_1+1)}}{1 - p_1^{-s}} \frac{1 - p_2^{-s(N_2+1)}}{1 - p_2^{-s}} \ldots \frac{1 - p_l^{-s(N_l+1)}}{1 - p_l^{-s}}.$$

In the limit $N \to \infty$ it gives the Euler infinite product representation of zeta function

$$\zeta(s) = \prod_{primes} \frac{1}{1 - \frac{1}{p^s}},$$

as the Planck formula for gas of prime numbers.

Keywords Riemann zeta function · quantum calculus · Planck formula · prime numbers · Berry-Keating Hamiltonian

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BOUNDEDNESS AND MAPPING PROPERTIES OF COMMUTATORS OF B-MAXIMAL OPERATORS

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ABSTRACT

This study focuses on the contemporary harmonic analysis associated with the generalized translation operator and the Laplace-Bessel differential operator within the framework of variable exponent Lebesgue spaces. In recent years, non-standard function spaces, particularly variable Lebesgue spaces, have attracted significant interest due to their applications in fluid mechanics, elasticity, and variational calculus. In this work, we investigate the structural and mapping properties of maximal sublinear operators generated by the Laplace-Bessel operator. Specifically, we establish the boundedness of both the commutator of the B-maximal operator and the B-maximal commutator itself on the variable exponent Lebesgue spaces $L_{p(\cdot),\gamma}(\mathbb{R}^n_{k,+})$. Under appropriate regularity conditions on the variable exponent $p(\cdot)$, such as the standard local log-Hölder continuity and suitable decay conditions at infinity, and assuming that the local symbol function b belongs to the generalized Bounded Mean Oscillation space BMO_γ, we prove the continuous embedding and robust boundedness results for these non-linear operator structures. Furthermore, the corresponding sharp B-maximal operator is introduced and evaluated to provide a complete characterization of the underlying space geometry. The pointwise estimates and innovative decomposition techniques utilized in our proofs allow us to overcome the difficulties arising from the non-invariant nature of the variable exponent under the action of the generalized translation operator T^y. Our findings provide a comprehensive, natural, and non-trivial extension of classical maximal commutator estimates to the variable exponent Laplace-Bessel setting, thereby filling a significant gap in the weighted operator theory on non-standard spaces.

***K*eywords** Bounded mean oscillation space · commutator · maximal operator · variable Lebesgue space

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TEACHING LINEAR PROGRAMMING THROUGH REAL CONSTRUCTION OPTIMIZATION CASES

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ABSTRACT

The teaching of Operations Research, and specifically Linear Programming, within engineering mathematics curricula has traditionally relied on abstract problem formulations and textbook exercises that bear little resemblance to the complex, multi-variable challenges encountered in real professional practice. While students are able to reproduce algorithmic procedures such as the Simplex method under examination conditions, they frequently struggle to recognize the applicability of these tools in authentic engineering contexts. This disconnect between mathematical formalism and practical relevance represents one of the most persistent challenges in applied mathematics education at technical universities, contributing to reduced student motivation, surface-level learning, and limited transfer of knowledge to professional settings. This paper presents the design and implementation of a case-based teaching approach for Linear Programming, developed and tested within the Applied Mathematics curriculum at the Technical University of Civil Engineering Bucharest (UTCB). The central pedagogical proposition is that real construction optimization cases can serve as powerful teaching vehicles, simultaneously anchoring abstract mathematical concepts in recognizable engineering scenarios and stimulating active student engagement. By situating the Simplex method and linear optimization within problems drawn directly from civil engineering practice, students are encouraged to move beyond procedural competency toward conceptual understanding and applied mathematical reasoning. The study introduces three carefully designed construction cases used as the primary teaching medium. The first case addresses the optimal allocation of construction materials across multiple building sites under budget and supply constraints, requiring students to formulate and solve a multi-variable linear programming problem reflecting realistic procurement conditions. The second case involves the minimization of transportation costs for raw materials delivered to construction sites, modeled as a classical transportation problem and solved using both the Simplex method and the Northwest Corner method. The third case focuses on the optimal scheduling of workforce allocation across concurrent project phases, incorporating resource availability constraints and deadline requirements typical of civil engineering project management. Each case is accompanied by full mathematical formulation, step-by-step solution procedures, and a discussion of how variations in real-world parameters affect the optimal solution, thereby reinforcing sensitivity analysis as an essential component of applied linear programming. The implementation was carried out with two parallel student cohorts enrolled in the Operations Research course at UTCB. One cohort followed the traditional instruction model based on standard textbook exercises, while the second cohort engaged exclusively with the real construction cases described above. Student performance was evaluated through a common mid-semester assessment covering problem formulation, solution procedure, and interpretation of results. In addition, a structured feedback questionnaire employing a five-point Likert scale was administered to measure student perceptions of relevance, motivation, and confidence in applying linear programming to engineering problems. Attendance rates across the intervention period were also recorded as a secondary indicator of engagement. The results indicate that students exposed to real construction cases demonstrated measurably stronger performance in problem formulation and solution interpretation, while reporting

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significantly higher levels of motivation and perceived relevance compared to the control cohort. Notably, the case-based group exhibited greater ability to adapt the mathematical model when problem parameters were modified, suggesting deeper conceptual understanding rather than procedural imitation. These findings support the integration of authentic engineering cases as a core pedagogical strategy in Operations Research teaching at technical universities. The paper concludes with recommendations for curriculum designers and mathematics educators seeking to embed real-world optimization problems into Linear Programming courses, and discusses the conditions under which case-based teaching is most effective. Limitations of the study and directions for future research, including the extension of this approach to integer programming and multi-objective optimization, are also addressed.

***K*eywords** linear programming · operations research · case-based teaching · engineering mathematics · Simplex method · construction optimization · active learning

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GEODESICS EQUATIONS OF SPACE–LIKE HELICOIDAL SURFACES IN MINKOWSKI SPACE–TIME

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ABSTRACT

Minkowski space–time serves as the geometric setting of special relativity and has been extensively studied from both mathematical and physical perspectives. Among the various surface classes in this space, helicoidal surfaces attract considerable attention because of their symmetry properties and rich geometric behavior. In this talk, we study the geodesics of space–like helicoidal surfaces in Minkowski space–time. Using the Euler–Lagrange equations, we derive the integral equations characterizing these geodesics. We also present several explicit examples via Mathematica. As some of the resulting integrals are difficult to evaluate analytically, Simpson's formula is employed to obtain numerical approximations.

***K*eywords** Geodesics · Euler–Lagrange equation · Helicoidal surface · Minkowski space–time

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DISTRIBUTION OF FEASIBLE SOLUTIONS FOR APPROXIMATE BLOCK DIAGONALIZATION WITH PENALTY COEFFICIENTS IN CLASSICAL, PSEUDO-QUANTUM AND QUANTUM ANNEALING

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ABSTRACT

One example of matrix computations, which is very important in science and engineering, is eigenpair computation. Recently, the Block Jacobi method, which possesses excellent parallelism, has attracted considerable attention as one of the good methods for calculation of eigenpairs. The method divides the input matrix into square blocks and repeatedly annihilates the off-diagonal blocks by orthogonal transformations to gradually bring the matrix to a diagonal form [1]. It has been confirmed that the convergence of the method can be accelerated by introducing preprocessing known as Approximate Block Diagonalization (ABD), which maximizes the sum of squared Frobenius norms of diagonal blocks by symmetric permutation of rows and columns, which does not change the eigenvalues of the target matrix [2].

Since ABD is a combinatorial optimization problem, a recent research attempted to solve it by quantum annealing [3]. In general, annealing methods cannot deal with constraints directly, so the penalty method is used to incorporate the constraints. However, determining the best penally parameter is not easy, especially when there are two or more constraints as in the ABD problem. In this research, in addition to quantum annealing, we applied classical annealing and pseudo-quantum annealing to the ABD problem and investigated the influence of the two penalty parameters by grid search. In particular, we focused on two kind of indices, namely, the number of feasible solutions and the maximum objective function value, as functions of the two parameters and studied how they exhibit different behaviors among the three annealing methods. Finally, we propose a method to select the penalty parameters automatically for the three annealing methods.

Keywords Block diagonalization · Annealing · QUBO · Unconstrained optimization · Penalty method · automatic parameter tuning

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A DETERMINISTIC MODEL OF STOMATAL CONDUCTANCE IN C3 PLANTS EXPLICITLY INCORPORATING LEAF TEMPERATURE

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ABSTRACT

Extreme climatic events, such as heat waves and prolonged droughts, have intensified with climate change, directly affecting metabolic processes of C3 plants, such as photosynthesis and transpiration. These processes are associated with the functioning of stomata, microscopic pores located on the leaf surface that control CO_2 uptake and water vapor loss, processes quantified through stomatal conductance (g_s) [3, 4]. Under these conditions, the leaf-to-air vapor pressure deficit (VPD_l) and leaf water balance are directly modified [1, 2]. In this context, leaf temperature (T_l), which results from the continuous exchange of energy between the plant and its environment, represents the thermal state of the plant and constitutes a relevant variable for describing the plant response to environmental stress. In addition, T_l directly contributes to the estimation of VPD_l, since this deficit depends on the difference between the saturation vapor pressure at leaf temperature and the actual vapor pressure of the air [1].

Despite the importance of stomata in leaf thermoregulation, the specific stomatal responses to temperature and their influence on the dynamics of g_s remain a matter of debate [5]. Several studies have reported contrasting behaviors: while some indicate an increase in g_s with rising temperature, even under constant VPD_l [1], others highlight a decoupling between photosynthesis and stomatal conductance g_s under heat and drought conditions [5]. This variability reflects the complexity of the phenomenon and explains why the relationship between T_l, VPD_l, and g_s is still not fully understood.

Therefore, it is important to develop quantitative models that explicitly integrate T_l into the dynamics of g_s, considering the thermal and physiological mechanisms that can improve the prediction of stomatal behavior under global warming scenarios [6, 7, 8]. In this work, a dynamic model of g_s is formulated and analyzed for C3 plants, explicitly incorporating T_l and VPD_l. The model is formulated through a first-order ordinary differential equation for $g_s(t)$, considering different time constants for stomatal opening and closing processes. The target function is constructed from factors associated with photosynthetic photon flux density (Q), T_l, and VPD_l, in order to represent equilibrium conductance under environmental conditions. The analysis focuses on qualitative properties of the model, including existence and uniqueness of solutions. This formulation provides a mathematical basis for studying the transient response of g_s to environmental changes.

Keywords Stomatal conductance · Dynamical systems · Ordinary differential equations · Asymptotic stability · Leaf temperature

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AN INTRODUCTION TO BLAISE AND BLAISE-LUCAS VECTORS

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ABSTRACT

Integer sequences have been extensively studied due to their rich structure and numerous applications. New approaches have been developed to the study of number sequences, with particular emphasis on their vector representation, as seen, for example, in [2], where Çetinberk and Yüce studied the vector products of the Fibonacci 3-vectors, 4-vectors and 7-vectors; Bingöl and Gökbaş, in [1], presented the inner product, norm and vector products for the Horadam vectors.

In this paper, we devote our attention on the investigation of Blaise numbers and Blaise–Lucas numbers sequences. These number sequences were presented by Soykan in [3], where several of their properties were introduced, in particular the relations between Blaise, Blaise-Lucas and Adjusted Jacobsthal-Padovan, Jacobsthal-Perrin (Jacobsthal-Perrin-Lucas), Jacobsthal-Padovan, and modified Jacobsthal-Padovan numbers. Uysal et al in [4] studied a quaternionic sequence using these types of numbers. The aim of this work is to introduce the vector representations of Blaise and Blaise–Lucas numbers and study some of its properties.

***K*eywords** Blaise numbers · Blaise-Lucas numbers · Blaise n-vector · Blaise-Lucas n-vector · linear algebra

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MAPPING CANDIDATE ATTRITION ACROSS A TECHNICAL INTERNSHIP ASSESSMENT PIPELINE

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ABSTRACT

The growing demand for technically proficient graduates in the information technology sector has intensified competition in internship recruitment, prompting organizations to adopt multi-stage assessment pipelines designed to filter candidates efficiently. While such pipelines are widely used, the patterns of candidate attrition across successive stages remain poorly documented in academic literature, particularly in the context of Eastern European higher education markets. Understanding where and why candidates disengage, whether due to lack of preparation, self-selection, or structural barriers, is essential for designing fairer and more effective selection processes. This study aims to quantify and analyse candidate drop-off across a two-stage technical assessment pipeline used in the selection of technical developer interns (specifically targeting PL/SQL programming competencies), and to identify systematic patterns related to institutional affiliation, academic level, and domain of study. The analysis draws on a recruitment dataset comprising 456 applicants from three major Romanian universities: the University Politehnica of Bucharest (UPB), the Academy of Economic Studies (ASE), and the University of Bucharest (UNIBUC), spanning undergraduate years B2 through B4 and master's years M1 and M2. The pipeline consisted of two sequential instruments: a general cognitive and aptitude screening via TestGorilla, followed by a domain-specific technical assessment via Getik. Candidate outcomes were classified across four distinct states, namely a failed attempt (FAIL), assigned to candidates who did not engage with the assessment at all, a valid numerical performance score (SCR), a did-not-complete status (DNC), assigned to candidates who started but did not finish the assessment, and a retired status (RET), assigned to candidates who withdrew during the stage, enabling funnel-level attrition analysis at each transition point of the pipeline. Attrition rates were computed as stage-level conversion ratios and compared across institutional and cohort dimensions using descriptive and comparative statistical analysis. The data reveal substantial and progressive attrition throughout the pipeline. At the TestGorilla stage, 31.6% of applicants recorded a zero score indicating non-participation, while an additional 13.2% did not complete the test, leaving only 55.3% of the total pool with a positive score and a mean of 57.2 points. Progression to the second stage was highly selective, as only 79 candidates were invited to the Getik assessment, of whom 22 did not complete it and 9 were retired, yielding 48 candidates with a valid technical score, representing a final conversion rate of 10.5% of the original applicant cohort. The mean score among successful Getik participants rose to 64.5 points, consistent with a positive selection effect across stages. Analysis by academic year reveals that B3 students exhibited the highest first-stage pass rate at 64%, compared to 50 to 53% for B2, B4, and M1 cohorts. At the institutional level, candidates from UPB's Faculty of Automatic Control and Computer Science and UNIBUC's Faculty of Mathematics and Informatics demonstrated stronger pipeline progression, while ASE-affiliated profiles showed higher rates of non-completion at the technical validation stage. The findings indicate that the dominant form of attrition in this pipeline is early-stage disengagement rather than failure under assessment conditions, a distinction with significant practical implications for recruitment design. The high non-attempt rate at the TestGorilla stage suggests that

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a substantial proportion of applicants may apply without genuine commitment or adequate technical preparation. Furthermore, the steep drop between the two assessment stages, from 252 candidates with a valid first-stage score to only 48 with a confirmed technical result, points to a structural bottleneck that may warrant redesign through threshold calibration, intermediate self-assessment instruments, or targeted pre-application guidance. These results contribute empirical evidence to the growing literature on recruitment analytics and talent pipeline optimization, and offer actionable recommendations for IT organisations seeking to improve internship selection efficiency without compromising technical quality standards.

***K*eywords** internship recruitment · candidate attrition · technical assessment pipeline · PL/SQL · funnel analytics · higher education

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NON-AUTONOMOUS EULER–LAGRANGE ODES VIA TEMPERED RIEMANN–LIOUVILLE FRACTIONAL-DERIVATIVE-TYPE VARIATIONS

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ABSTRACT

In this paper, we provide new contributions to variational analysis and optimal control theory by addressing a new class of optimal control models based on tempered Riemann– Liouville fractional-derivative-type variations. Specifically, we formulate an optimal control problem equipped with a C^1-class controlled Lagrange functional, a simple integral objective functional, and a set of ordinary differential equations (ODEs) as first-order constraints. Thereafter, we recall appropriate variational techniques and design the perturbed functions using tempered Riemann– Liouville fractional derivatives of order $\alpha \in (n-1, n], n \in \mathbb{N}$. Notably, we reconsider the integration-by-parts formula within a fractional setting. In this context, we develop a variational framework yielding the non-autonomous Euler-Lagrange ODEs. To the best of our knowledge, this is the first study to incorporate fractional-derivative-type variations. As a result, the inclusion of tempered fractional-derivative-type variations leads to a broader class of Euler-Lagrange equations, capable of capturing memory and nonlocal effects that are not accounted for by the classical variational framework.

***K*eywords** Tempered fractional derivatives · Non-autonomous Euler–Lagrange equations · Optimal control theory · Riemann–Liouville fractional calculus

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NECESSARY OPTIMALITY CONDITIONS FOR DISCRETE SYSTEMS DESCRIBED BY GRADIENT-TYPE HAMILTONIAN

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ABSTRACT

The present study introduces a modern variational framework for processes characterized by discrete evolution. Within a class of perturbed functions governed by gradient-type variations, we investigate the necessary optimality criteria for discrete multidimensional optimal control problems involving higher-dimensional recurrence relations. First, we extend several concepts from the continuous-time setting to the discrete framework. Concretely, first-order Lagrangians, classical differential operators, complete controllability conditions and equality-type constraints are reconsidered through discrete formulations. Thereafter, we formulate the model under study by incorporating first-order higher-dimensional iterative processes together with discrete gradient-type variations. Our main results establish a discrete Pontryagin maximum principle and derive the associated Hamiltonian-based necessary optimality conditions. Notably, the introduction of gradient-type variations leads to partial differential equations whose right-hand side is governed by a real constant, yielding a more comprehensive and versatile description of the system's dynamics, as these equations incorporate structural information beyond what the classical formulation can capture.

Keywords Discrete optimal control · Multi-directional recurrence relations · Discrete gradient-type variations · Pontryagin maximum

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A Comparative Study of Classical and Quantum Support Vector Machine for Diabetic Foot Classification Using the Cluster Thermal Index

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Abstract

Diabetes mellitus is one of the most common chronic diseases in the world and is frequently associated with severe complications such as diabetic foot ulcers, which can lead to infection, hospitalization, and lower-limb amputation if not detected early. Infrared thermography has emerged as a promising non-invasive imaging modality for diabetic foot assessment, as changes in plantar temperature distributions can detect physiological abnormalities before they become visible clinical symptoms. Mathematical descriptors and machine learning techniques are increasingly used to characterize thermographic patterns and improve classification performance [3, 5].

Recent advancements in quantum computing have prompted the adaptation of well-known machine learning algorithms to quantum computational frameworks, resulting in Quantum Machine Learning and quantum-inspired learning approaches. These methods draw inspiration from principles of quantum mechanics, such as superposition and quantum state representations, to construct alternative data encodings and feature spaces. In particular, Quantum Support Vector Machine (QSVM) extend classical Support Vector Machine (SVM) through the use of quantum feature maps and quantum kernel functions, enabling data to be represented in Hilbert spaces generated by quantum circuits. From a mathematical perspective, these representations provide alternative formulations of kernel-based classification and may offer new possibilities for modelling complex data structures. As a result, quantum and quantum-inspired learning methods have attracted increasing attention in areas such as medical image analysis and pattern recognition [2, 4].

In this context, this paper proposes a comparative analysis between classical SVM and QSVM for diabetic foot classification using engineered thermographic features. The methodology is based on the Cluster Thermal Index (CTI), a mathematical descriptor that quantifies the deviation of an individual's plantar thermal profile from that of a healthy reference population through temperature clustering, together with additional global and regional statistical features extracted from thermograms [1]. These descriptors define a common feature space for both classical and quantum classifiers, enabling a direct comparison under equivalent conditions.

The performance of both approaches is measured using standard classification metrics such as accuracy, precision, recall, F1-score, specificity, and area under the ROC curve. The purpose of the study is to evaluate the applicability of the QSVM to CTI-based diabetic foot classification and to investigate its performance relative to the classical SVM in thermographic biomedical analysis. In addition to contributing to diabetic foot assessment and computer-aided diagnosis, the findings contribute to the emerging field of quantum machine learning in healthcare, emphasizing the importance of mathematical feature engineering for infrared thermography-based diagnostic systems.

Keywords Cluster Thermal Index · Kernel Methods · Quantum Machine Learning · Quantum Support Vector Machine · Support Vector Machine · Infrared Thermography · Diabetic Foot

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A QUANTUM-INSPIRED SEGMENTATION ALGORITHM APPLIED TO THERMOGRAPHIC FOOT IMAGES

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ABSTRACT

The usage of medical image analysis in modern healthcare is an indispensable and irreversible process, covering a wide range of phases from early diagnosis to surgery planning [6]. Within this field, infrared thermography has been successfully established as a rapid, non-contact, and non-invasive technique to visualize the temperature distribution of diabetic-foot related problems [3].

Classical machine learning and deep learning approaches have immensely enhanced the capability of segmentation, classification and image processing. Recently, more complex AI-driven and deep learning algorithms have intensified this process. However, these conventional computational methods face critical bottlenecks, namely high computational costs and an absolute dependency on massive, heavily labeled training datasets that are rarely available in specialized medical fields [2, 6].

Quantum computing emerged as a promising route to overcome these limitations of classical computing. It operates on the principles of quantum mechanics, offering new computational paradigms such as the use of qubits, superposition, massive parallelism, and quantum gates and circuits that operate under unitary transformations [6]. However, the limited availability of such hardware in the current Noisy Intermediate-Scale Quantum era, its operational unfeasibility, and the extreme infrastructure costs involved make pure quantum computing still not a viable option for immediate clinical application [1, 6].

Quantum inspired algorithms are designed to mathematically simulate quantum mechanics principles but operate entirely on conventional classical devices with the aim of achieving and exceeding the performance of previously known classical methods [4, 7]. They can learn with fewer parameters and lower sample requirements than their classical counterparts in some instances [6, 7], enhancing performance and being better cost-efficient in training and execution, emerging as a viable alternative to overcome both classical and quantum hardware boundaries.

This paper proposes the application of a quantum-inspired clustering algorithm focused on automated image segmentation for plantar thermograms aimed at the detection of diabetes anomalies. By dividing the plantar region into independent clinical angiosomes [5] without requiring massive datasets, the proposed algorithm offers a mathematical tool for localized thermal analysis [3, 4], paving the way for data-efficient, automated monitoring of diabetic foot pathologies.

***K*eywords** Quantum-Inspired Algorithms · Image Segmentation · Clustering · Infrared Thermography · Diabetic Foot

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NETWORK CHARACTERIZATION OF TSUNAMI TIME SERIES USING VISIBILITY GRAPH ANALYSIS: THE 30 OCTOBER 2020 İZMİR–SAMOS TSUNAMI EVENT

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ABSTRACT

The 30 October 2020 İzmir–Samos tsunami generated substantial sea-level oscillations throughout the eastern Aegean Sea and was recorded at several tide-gauge stations. Since the event, numerous studies have examined its hydrodynamic behavior, spectral characteristics, and predictability using conventional time-series analysis techniques [1]. However, comparatively less attention has been given to the structural organization of the recorded signals and the possibility of describing tsunami evolution from a network perspective. Sea-level records from the Syros and Bodrum tide-gauge stations are analyzed using visibility graph analysis, a method that transforms a time series into a complex network by establishing connections between mutually visible observations [2]. Following mean removal and spectral detrending, natural visibility graphs are constructed from the processed tsunami records. The resulting network representation enables the characterization of the time series through graph-theoretical measures rather than solely through conventional amplitude- or frequency-based descriptors. The analysis focuses on whether different phases of tsunami evolution can be distinguished through the topological properties of visibility graphs constructed from sea-level records. To examine this issue, each record is divided into three periods corresponding to pre-tsunami conditions, the interval of strongest tsunami activity, and the subsequent oscillatory stage. For each period, network measures including average degree, clustering coefficient, and average path length are evaluated and compared. Rather than proposing a new graph construction algorithm, the study focuses on the applicability of visibility graph analysis to tsunami observations. The resulting network measures provide a complementary description of tsunami-induced sea-level fluctuations and support the use of network-based approaches in coastal and ocean engineering research.

Keywords Tsunami time series · Visibility graph analysis · Complex networks · Graph theory · İzmir–Samos Tsunami

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A New Generalization of Frank Matrix

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Abstract

In this presentation, we introduce a novel two-parameter generalization of the well-known Frank matrices and their Hadamard inverse, examine their linear algebraic properties, and present the results with numerical examples.

***K*eywords** Frank Matrix · Special Matrices · Matrix Factorization

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DESIGN PARAMETERS OF OSCILLATION ELIMINATOR

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ABSTRACT

The paper addresses the mathematical modelling of advanced vibration systems. It focuses on the analysis and modification of existing mathematical models, and discusses the development of new ones. These models will be intended to meet the experimental requirements of developing progressive dynamic dampers for mechanical systems. The primary objective is to identify the key design parameters governing the damping and overall system's response. The theoretically determined identifiers will be then verified and assessed within the experiments. The design of innovative progressive dampers typically relies on a combination of dynamic modelling and experimental validation.

Effective damping can significantly extend machine life, reduce the negative impacts of vibration on operation, improve human comfort, increase energy efficiency, and lower environmental impact.

Practically, the detected parameters will be applied in the construction of inventive vibration systems and their control. On the other hand, we plan to adapt the mathematical model also for academic use and include it directly into education of future engineering specialists. Moreover, this can serve as excellent material for active and interactive learning methodologies. These approaches effectively foster teamwork, a soft skill highly sought after by employers due to a currently widely recognized shortage of collaborative and communication abilities. Ultimately, it strengthens students' comprehensive engineering competencies. We also expect that the integration of theory, experiment, and practical application will boost students' engagement with the mathematical fundamentals of engineering as well.

***K*eywords** Mechanical vibration system · Mathematical model · Advanced damper

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REMARKS ON A GENERALIZED JOUKOWSKI TRANSFORMATION

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ABSTRACT

The classical complex Joukowski transformation is a conformal mapping that maps a disk onto an airfoil denoted as Joukowski airfoils. Its properties and role in physical applications have motivated the study of its generalization. However, extending the Joukowski function to higher-dimensional contexts has reveal some intrinsic difficulty, for example, the fact that in higher-dimensional singularities, folds, or self-intersections can be created. In [2], a higher-dimensional analogue of the Joukowski transformation is described using hypercomplex Appell polynomials introduced in [4], and the different geometric configurations of the image of spheres where studied for the tree-dimensional case. The main topic of [1] also focused on three-dimensional generalized Joukowski transformation of order two, and their geometric deformations were thoroughly explored through visualizations. In [3], was shown that, for certain radii, the three-dimensional generalized Joukowski transformation presented in [2] is 4-quasiconformal.

In this work we intend to revisit the generalized Joukowski-type transformation and study some of its properties.

***K*eywords** Hypercomplex differentiable functions · Generalized Joukowski transformation · Quasiconformal mappings

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HIGHER-ORDER RECURRENCES IN SCIENCE AND ENGINEERING EDUCATION: FIBONACCI AND TRIBONACCI APPROACHES TO ALGORITHMIC THINKING

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ABSTRACT

Linear recurrence sequences constitute an important class of mathematical objects, with applications in mathematics, computer science, and engineering. They provide canonical models for discrete dynamical systems and play a central role in the study of recursive processes, difference equations, and discrete structures. This work revisits the Fibonacci sequence, introduced by Leonardo of Pisa in the Liber Abaci (1202), and its higher-order generalisation, the Tribonacci sequence, formalised in the twentieth century, as a unifying framework that articulates mathematical foundations, modern applications and educational practice. We review the recurrence relations, characteristic equations and Binet-type closed forms of both sequences, and discuss their matrix representations, in which the golden ratio and the Tribonacci constant appear as the dominant eigenvalues of the corresponding 2×2 and 3×3 companion matrices, respectively. Building on these algebraic foundations, we propose a structured didactic pathway - from the classical rabbit problem to the generalised Tribonacci model and its algorithmic implementation - designed to foster algorithmic and computational thinking in mathematics, science and engineering education. The transition from second- to third-order recurrence is presented not as a mere algebraic extension, but as a conceptual leap supporting modelling competencies, abstraction and recursive reasoning, with direct relevance for the training of future scientists and engineers.

***K*eywords** Fibonacci sequence · Tribonacci sequence · higher-order linear recurrences · Matrix representation · Algorithmic thinking · Mathematics education for science and engineering

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DYNAMICS AND STABILITY OF MULTIPLICATIVE FIBONACCI-TYPE RECURRENCE

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ABSTRACT

An interesting variation of the Fibonacci sequence arises when each new term is defined as the product, rather than the sum, of the two preceding terms. In this work, a generalised multiplicative Fibonacci-type recurrence is introduced. For selected particular cases, closed-form expressions for the general term are derived. The recurrence is then reformulated as a two-dimensional discrete dynamical system, allowing for the analysis of the existence of fixed points and the study of their local stability. However, under appropriate positivity conditions, the nonlinear structure becomes considerably simpler after a logarithmic renormalisation, by which the multiplicative recurrence is transformed into an additive, Fibonacci-type one. Within this framework, an explicit representation for the renormalised dynamics is derived through a Fibonacci-type variation-of-constants formula. The asymptotic behaviour of three particular cases is included to illustrate the theoretical results. The proposed framework provides a unified perspective on multiplicative analogues of Fibonacci-type sequences and suggests further extensions to other nonlinear recurrences of similar structure.

***K*eywords** Multiplicative Fibonacci sequence; perturbation; stability; recurrence relation.

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ACTIVE LEARNING APPROACHES IN NUMERICAL METHODS FOR CIVIL ENGINEERING STUDENTS

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ABSTRACT

The teaching of Numerical Methods within engineering mathematics curricula has long been dominated by lecture-based instruction focused on algorithmic exposition and formal mathematical derivations. While this approach ensures theoretical rigor, it frequently fails to convey the practical relevance of numerical techniques to civil engineering students, who often struggle to connect iterative computational procedures with the structural, geotechnical, and thermal problems they encounter in their professional training. This disconnect contributes to reduced student motivation, passive learning behaviors, and limited capacity to apply numerical tools independently in authentic engineering contexts. This paper presents the design, implementation, and evaluation of active learning strategies integrated into a Numerical Methods course delivered to civil engineering students at the Technical University of Civil Engineering Bucharest (UTCB). The central argument is that active learning methodologies, when carefully aligned with the mathematical content of numerical analysis and grounded in recognizable civil engineering applications, can significantly improve both conceptual understanding and student engagement. The study is grounded in constructivist and student-centered learning theory, which positions the learner as an active constructor of knowledge rather than a passive recipient of algorithmic procedures. Three active learning strategies were designed and implemented within the course structure. The first strategy applied project-based learning to core numerical topics including linear system solving, polynomial interpolation, and finite element approximations, using authentic problems drawn from structural analysis and materials resistance typical of civil engineering practice. Student teams were required to formulate the engineering problem mathematically, select and implement an appropriate numerical method, and interpret the results within realistic engineering constraints. The second strategy employed problem-based learning centered on numerical solutions of ordinary and partial differential equations, with applications in thermal conduction through building materials and soil settlement modeling. Students were presented with open-ended, ill-structured problems that required iterative refinement of both the mathematical model and the numerical approach. The third strategy integrated computational tools, specifically Python, MATLAB, and GeoGebra, as active exploration environments in which students could experiment with algorithm parameters, visualize convergence behavior, and compare the performance of alternative numerical schemes in real time. The implementation was carried out with two parallel student cohorts enrolled in the Numerical Methods course at UTCB. One cohort followed the conventional instructional model based on formal derivations and standard textbook exercises, while the second cohort engaged with the active learning strategies described above. Student performance was assessed through a common written evaluation covering problem formulation, method selection, computational accuracy, and result interpretation. A structured feedback questionnaire employing a five-point Likert scale was administered to both cohorts to measure perceptions of relevance, motivation, confidence, and conceptual clarity. Attendance rates across the intervention period were recorded as a supplementary indicator of engagement. The results indicate that students

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in the active learning cohort demonstrated stronger performance in problem formulation and result interpretation, while showing significantly higher levels of motivation and perceived relevance of numerical methods to civil engineering practice. Notably, these students exhibited greater flexibility in selecting and adapting numerical methods when problem conditions were varied, suggesting deeper procedural and conceptual understanding compared to the control cohort. These findings support the systematic integration of active learning strategies into Numerical Methods courses at technical universities, particularly when instructional design explicitly connects mathematical content to authentic engineering scenarios. The paper concludes with recommendations for mathematics educators and curriculum designers seeking to embed active learning into Numerical Methods teaching, and discusses the institutional and logistical conditions under which such approaches are most effective. Limitations of the present study and directions for future research, including the extension of active learning strategies to advanced topics such as finite difference methods, numerical optimization, and stochastic simulation, are also addressed.

***K*eywords** numerical methods · active learning · civil engineering education · project-based learning · computational tools · engineering mathematics

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ON FIBONACCI q-MULTIPLE ZETA VALUES

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ABSTRACT

We introduce a finite q-analogue of multiple zeta values involving modified Fibonacci polynomials. This family provides a natural extension of the classical multiple zeta values, recovering them in the limiting case as $q \to 1$ and the truncation parameter tends to infinity. We establish explicit forms for several classes of these finite q-series at roots of unity and investigate their arithmetic properties.

***K*eywords** q-multiple zeta values · Fibonacci polynomials

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BIFURCATION STRUCTURES AND CHAOTIC DYNAMICS IN A FRACTIONAL-ORDER RICHARDS MODEL WITH ENVIRONMENTAL FEEDBACK

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ABSTRACT

In this study, a fractional-order population model based on the modified Richards growth law is proposed to investigate the combined effects of environmental feedback and the Allee effect on population dynamics. The model incorporates a density-dependent carrying capacity and a critical population threshold, providing a more realistic description of ecological growth processes. Since low-dimensional continuous and fractional-order systems cannot generate chaotic behavior, the piecewise constant argument (PCA) approach is employed to derive a discrete-time counterpart of the proposed model.

The existence and local stability of the equilibrium points are analyzed by means of the Schur–Cohn criterion. The results show that the discrete model exhibits a Flip bifurcation near the positive equilibrium point E_1 and a Neimark–Sacker bifurcation near the positive equilibrium point E_2 as the system parameters pass through the corresponding critical thresholds. The direction and stability of the bifurcating solutions are determined using center manifold theory and normal form analysis. Numerical bifurcation diagrams and phase portraits reveal the emergence of periodic, quasi-periodic, and chaotic dynamics. The presence of chaos is further confirmed through the computation of the maximum Lyapunov exponents.

To suppress undesirable chaotic oscillations, state-feedback and hybrid control strategies are implemented. Numerical simulations demonstrate that both control approaches successfully stabilize the system and enlarge the parameter regions corresponding to regular dynamics. The obtained results indicate that environmental feedback and the Allee effect significantly influence the stability boundaries and bifurcation structure of the model, leading to a rich variety of dynamical outcomes.

***K*eywords** Bifurcation · Chaos · Mathematical model · Stability · Fractional order

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COEFFICIENT BOUNDS AND INCLUSION PROPERTIES FOR A SUBCLASS OF ANALYTIC FUNCTIONS DEFINED BY KOMATU INTEGRAL OPERATOR

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ABSTRACT

The present investigation in Geometric Function Theory of Complex Analysis is essentially motivated by the importance and usefulness of the coefficient bounds and the coefficient estimates for functions belonging to several analytic and univalent function classes, such as the classes of starlike and convex functions. In this paper, we introduce and study a new subclass of analytic functions defined by Komatu integral operator in the unit disk. For this subclass, coefficient bounds and inclusion properties are obtained.

Keywords Analytic functions · Komatu integral operator · Coefficient bounds · inclusion properties

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ON PARTIAL SUMS OF THE MODIFIED TRICOMI FUNCTION

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ABSTRACT

The aim of the present investigation is to establish lower bounds for the modified Tricomi function and for its sequences of partial sums. The modified Tricomi function is an important special function that appears in several areas of mathematical analysis, particularly in the study of analytic functions and geometric function theory. Since partial sums provide finite approximations of analytic functions, examining their behavior is essential for understanding whether important geometric properties are preserved under truncation. In this paper, we derive lower estimates for both the modified Tricomi function and its corresponding partial sums within the unit disk. Furthermore, we investigate a lower bound for the real part of the ratio of the derivative of the modified Tricomi function to the derivative of its sequence of partial sums. The obtained results contribute to the study of partial sums and provide useful refinements of related inequalities in analytic function theory.

***K*eywords** Tricomi function · Partial sums · Analytic functions

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THE FEKETE-SZEGÖ INEQUALITY FOR THE SUBCLASSES OF ANALYTIC FUNCTIONS RELATED WITH AN INTEGRAL OPERATOR

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ABSTRACT

Starlike and convex functions occupy a central position in the theory of analytic functions, and coefficient inequalities associated with these classes play a crucial role in understanding their geometric behavior. In this study, Fekete–Szegö type coefficient inequalities are established for classes of starlike and convex functions of complex order. Moreover, several corollaries are derived for particular choices of the parameters.

***K*eywords** Analytic functions · Komatu integral operator · Fekete-Szegö inequality

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NEIGHBORHOODS OF CERTAIN CLASSES OF ANALYTIC FUNCTIONS DEFINED BY THE MODIFIED TRICOMI FUNCTION

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ABSTRACT

By making use of the familiar concept of neighborhoods of analytic functions, the author proves several inclusion relations associated with the (n, δ) −neighborhoods of various subclasses defined by the modified Tricomi function. Special cases of some of these inclusion relations are shown to yield known results.

Keywords Analytic functions · Tricomi function · Neighborhoods

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TOPOLOGICAL SEMIGROUPS OF NONLINEAR OPERATORS IN UNIFORMLY CONVEX MODULAR SPACES

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ABSTRACT

This talk centers around the study of common fixed point properties of topological semigroups of nonexpansive mappings in modular vector spaces generalizing some results of Browder [2] for its Banach space counterpart. In the first part, we define a notion of uniform convexity for modular vector spaces which generalizes the classical concept of uniformly convex normed spaces, and study in this framework actions of (left) reversible semitopological semigroups of nonexpansive mappings on bounded closed convex sets. In the second part, we study such properties for arbitrary modular vector spaces for actions of topological right groups on bounded closed convex sets with normal structure in modular spaces without uniform convexity generalizing a result of Belluce and Kirk [1] in fixed point theory of Banach spaces.

***K*eywords** modular space · left reversible semigroup · uniform convexity

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PROPER RANDOM WALK SPLINE MODELS

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ABSTRACT

Splines are central to nonparametric regression, and intrinsic spline formulations such as the second-order random walk (rw2) have become standard for smoothing structured data. However, their intrinsic nature entails specific structural features—non-constant marginal variances, inflated boundary uncertainty, sensitivity to grid design, and unbounded predictive variance—that arise from the absence of a finite correlation range. While these characteristics serve the purpose of flexible smoothing, they can lead to unstable or misleading behavior in prediction or sparse-data settings when the intrinsic nature of the model is not carefully accounted for.

This thesis introduces the PRW2, a full-rank Gaussian Markov random field that serves as a proper counterpart to the intrinsic RW2 formulation. PRW2 preserves RW2's smoothing strength and computational efficiency while providing a coherent probabilistic structure with constant marginal variance, bounded predictive uncertainty, and stability under domain changes. Although RW2 and PRW2 belong to different model spaces, PRW2 closely reproduces the smoothing behavior of RW2 as the range parameter increases, ensuring smoothing behavior without requiring intrinsic specification.

. Bayesian Statistics . Gaussian Markov Random Fields (GMRF) . INLA . Spatio-temporal modeling

***K*eywords** Spline smoothing · INLA · Spatio-temporal modeling · Gaussian Markov Random Fields (GMRF) · Bayesian Statistics

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THE ALLEE EFFECT: ITS INFLUENCE ON A HOLLING-TANNER MODEL WITH GENERALIST PREDATORS

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ABSTRACT

The study of interactions between prey and predators is fundamental to mathematical ecology. Factors such as the Allee effect in prey and the availability of alternative food sources for predators significantly influence these dynamics. In this article, we analyze a Holling-Tanner model with an double Allee effect in prey and generalist predators, which is described by the autonomous ordinary differential equation system of Kolmogorov type. To simplify the calculations, a change of variables and a time rescaling were applied, resulting in a system topologically equivalent to the original, which allowed for a more detailed analysis. We establish the existence of a positively invariant region and show that the solutions are permanent. Moreover, we prove that the solutions are uniformly bounded. The conditions for the existence and stability of equilibrium points in the phase plane are established.

Keywords Allee effect · Bifurcation · Holling-Tanner model · Stability

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MATHEMATICAL ANALYSIS OF NONCE GENERATION IN MAVLINK-BASED UAV COMMUNICATION SYSTEMS

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ABSTRACT

Unmanned Aerial Vehicles (UAVs) rely on communication links to exchange telemetry, navigation data, and control commands between onboard flight controllers and ground control stations. In many UAV platforms this communication is implemented using the MAVLink protocol, a lightweight binary protocol designed for resource-constrained embedded systems and low-bandwidth links [1]. While MAVLink 2 introduces a message signing mechanism that provides authentication and integrity, it does not protect the confidentiality of transmitted data. Consequently, telemetry and control commands remain observable to adversaries capable of monitoring the communication channel.

To address this limitation, several works have proposed integrating cryptographic mechanisms into MAVLink-based communication stacks. In particular, lightweight authenticated encryption schemes are well suited to embedded UAV systems due to their low computational and energy requirements. Among these algorithms, Ascon-AEAD128 has recently been standardized by the National Institute of Standards and Technology (NIST) as the primary lightweight authenticated encryption algorithm for constrained devices [2]. The algorithm was specifically designed for environments such as embedded systems and Internet-of-Things platforms, where computational resources and power consumption are limited [3].

However, the security guarantees of nonce-based authenticated encryption schemes rely critically on the *nonce-respecting* assumption: the same nonce must never be reused with the same secret key. While this requirement is simple at the algorithmic level, it becomes significantly more subtle when the encryption primitive is integrated into an existing communication protocol. In particular, MAVLink packets include a sequence number field encoded on eight bits, which evolves according to modular arithmetic and necessarily wraps around after 256 transmitted packets. From a mathematical perspective, this behavior corresponds to the dynamics of a finite cyclic group and introduces deterministic periodicity in protocol state variables.

In this work we develop a mathematical framework for analyzing nonce generation in MAVLink-based communication systems using Ascon-AEAD128. Packet transmissions are modeled as a discrete state process in which the system state evolves with each transmitted packet. Nonce generation is represented as a deterministic function defined on the protocol state space, allowing the security requirement of nonce uniqueness to be formulated as an injectivity condition along the operational trajectory of the system.

Using this model, we analyze several natural nonce constructions derived from MAVLink packet fields. In particular, we show that constructions based directly on protocol sequence numbers lead to deterministic nonce collisions due to modular wraparound. Similar failures occur when the nonce is constructed by concatenating several protocol header fields, since some of these identifiers typically remain constant during a communication session. We also analyze random nonce generation and show that while deterministic collisions disappear, the probability of probabilistic collisions follows the classical birthday bound from applied cryptography [4].

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Finally, we study a session–counter construction in which the nonce is generated by combining a session identifier with a monotonically increasing counter. We prove that, under mild operational assumptions, this construction guarantees nonce uniqueness while remaining compatible with the constraints of UAV telemetry systems. The analysis highlights that the secure integration of authenticated encryption in communication protocols depends not only on the cryptographic primitive but also on the mathematical properties of the protocol state variables used to generate nonces.

The proposed framework provides a systematic approach for analyzing nonce generation in lightweight cryptographic systems and may serve as a basis for designing secure MAVLink-based UAV communication architectures.

***K*eywords** MAVLink protocol · Nonce generation · Lightweight cryptography

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GREATEST COMMON DIVISORS OF SHIFTED FIBONACCI AND LUCAS SEQUENCES

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ABSTRACT

Shifted Fibonacci sequences, obtained by adding a fixed integer a to the Fibonacci sequence, were first introduced in the literature in 1973. Although the greatest common divisor (gcd) of any two Fibonacci numbers is always a Fibonacci number, consecutive Fibonacci and Lucas numbers are relatively prime. Consequently, determining the gcd of consecutive shifted Fibonacci or Lucas numbers constitutes a special case worthy of investigation. Moreover, a variety of approaches, including different proof techniques and computational software packages, have been employed in the study of greatest common divisors. It was shown that consecutive shifted Fibonacci numbers are not necessarily relatively prime and that the values of $gcd(F_n + a, F_{n+1} + a)$ are unbounded when $a = \pm 1$. More recently, the gcd properties of shifted Fibonacci and Lucas numbers have been studied extensively, with particular emphasis on the cases $a = \pm 1$, ± 2, and ± 3. In this study, we investigate the greatest common divisors of consecutive shifted Fibonacci and Lucas numbers. Our results show that, unlike the case $a = \pm 1$, the greatest common divisors of consecutive shifted Fibonacci and Lucas numbers are bounded.

***K*eywords** Fibonacci numbers, Lucas numbers, shifted Fibonacci sequence, shifted Lucas sequence, greatest common divisor

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A GENERALIZATION OF (AMPLY) COFINITELY G-SUPPLEMENTED MODULES

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ABSTRACT

In this work, every ring has an identity and every module over a ring R is a unitary left R-module. Let M be an R-module and $U, V \leq M$. If $M = U + V$ and $M = U + T$ with $T \trianglelefteq V$ implies that $T = V$, or equivalently, $M = U + V$ and $U \cap V \ll_g V$, then V is called a g-supplement of U in M. M is said to be g-supplemented if every submodule of M has a g-supplement in M. M is said to be cofinitely g-supplemented if every cofinite submodule of M has a g-supplement in M. Let M be an R-module and $U \leq M$. If for every $V \leq M$ such that $M = U + V$, U has a g-supplement V' in M with $V' \leq V$, then we say U has ample g-supplements in M. If every submodule of M has ample g-supplements in M, then M is called an amply g-supplemented module. If every cofinite submodule of M has ample g-supplements in M, then M is called an amply cofinitely g-supplemented module. Let M be an R-module and T be a maximal submodule of M. If $T \trianglelefteq M$, then T is called a g-maximal submodule of M. The intersection of all g-maximal submodules of an R-module M is called the generalized radical (briefly, g-radical) of M and denoted by $Rad_g M$. If M have no g-maximal submodules, then the g-radical of M is defined by $Rad_g M = M$. Let M be an R-module and $U, V \leq M$. If $M = U + V$ and $U \cap V \leq Rad_g V$, then V is called a g-radical supplement of U in M. If every submodule of M has a g-radical supplement in M, then M is called a g-radical supplemented module. M is said to be cofinitely g-radical supplemented if every cofinite submodule of M has a g-radical supplement in M. Let M be an R-module and $U \leq M$. If for every $V \leq M$ such that $M = U + V$, U has a g-radical supplement V' in M with $V' \leq V$, then we say U has ample g-radical supplements in M. If every submodule of M has ample g-radical supplements in M, then M is called an amply g-radical supplemented module. If every cofinite submodule of M has ample g-radical supplements in M, then M is called an amply g-radical supplemented module. In this work, some new properties of (amply) cofinitely g-radical supplemented modules are investigated. It is clear that every (amply) cofinitely g-supplemented module is (amply) cofinitely g-radical supplemented.

Keywords Essential Submodules · Small Submodules · g-Radical · g-Supplemented Modules

2020 Mathematics Subject Classification: 13C13, 13C60.

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A GENERALIZATION OF BETA-STAR RELATION IN MODULES

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ABSTRACT

In this work, every ring has an identity and every module over a ring R is a unitary left R-module. Let M be an R-module. It is defined the relation $'\beta^{*\prime}$ on the set of submodules of an R-module M by $X\beta^{*}Y$ if and only if $Y+K=M$ for every $K\leq M$ such that $X+K=M$ and $X+T=M$ for every $T\leq M$ such that $Y+T=M$. Let M be an $R-$module and $K\leq V\leq M$. We say V lies above K in M if $V/K\ll M/K$. Let M be an R-module. It is defined the relation $'\beta_{g}^{*\prime}$ on the set of submodules of an R-module M by $X\beta_{g}^{*}Y$ if and only if $Y+K=M$ for every $K\trianglelefteq M$ such that $X+K=M$ and $X+T=M$ for every $T\trianglelefteq M$ such that $Y+T=M$. Let M be an R-module and $F\leq M$. A submodule K of M is called an F-small (or F-superfluous) submodule of M and written by $K\ll_{F}M$ if $T=M$ for every $T\leq M$ such that $F\leq T$ and $M=K+T$. A submodule T of M is said to be F-maximal if T is a maximal submodule of M and $F\leq T$. The intersection of all F-maximal submodules of M is called the F-radical of M and denoted by $Rad_{F}M$. If M have no F-maximal submodules, then the F-radical of M is defined by $Rad_{F}M=M$. Let M be an R-module and $F,U,V\leq M$. V is called an F-supplement of U in M if V is minimal with respect to $F\leq L\leq M$ and $M=U+L$. V is an F-supplement of U in M if and only if $M=U+V$ and $U\cap V\ll_{F}V$. If every submodule of M has an F-supplement in M, then M is called an F-suplemented module. Let M be an R-module and $F,U\leq M$. If for every $V\leq M$ such that $M=U+V$, U has a F-supplement $V^{'}$ with $V^{'}\leq V$, we say U has ample F-supplements in M. If every submodule of M has ample F-supplements in M, then M is called an amply F-supplemented module. Let M be an $R-$module and $F\leq M$. It is defined the relation $'\beta_{F}^{*\prime}$ on the set of submodules of the module M by $X\beta_{F}^{*}Y$ if and only if $Y+K=M$ for every $K\leq M$ such that $F\leq K$ and $X+K=M$ and $X+T=M$ for every $T\leq M$ such that $F\leq T$ and $Y+T=M$. In this work, some new properties of $'\beta_{F}^{*\prime}$ relation are investigated. $'\beta_{F}^{*\prime}$ relation is a generalization of $'\beta^{*\prime}$ relation and lying above in modules.

Keywords Modules · β^{*} relation · F-Small Submodules · F-Supplemented Modules

2020 Mathematics Subject Classification: 13C13, 13C60.

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ON THE DERANGEMENT-FIBONACCI POLYNOMIALS AND NUMBERS

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ABSTRACT

The generating function of derangement polynomials $D_n(x)$ is defined by

$$\frac{e^{-t}}{1-t}e^{xt} = \sum_{n=0}^{\infty} D_n(x)\frac{t^n}{n!} \tag{1}$$

for $n \in \mathbb{N}_0$ in [1].
When $x = 0$ is written in Equation (1), the derangement numbers D_n are obtained [1, 2].

In this study, derangement polynomials and derangement numbers are defined by means of Fibonomial coefficients. The generating functions of these polynomials and numbers are derived, and explicit formulas are obtained. Furthermore, several fundamental properties, including derivative and integral formulas, are established within the framework of Golden calculus. The results provide new insights into the interplay between derangement theory and Fibonomial coefficients and contribute to the development of Fibonacci-based special polynomial families.

***K*eywords** Derangement polynomials, · Fibonacci numbers · Golden calculus

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THE EXISTENCE OF AN UNBOUNDED SEQUENCE OF EIGENVALUES FOR A NONLINEAR TRANSMISSION PROBLEM

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ABSTRACT

Let $\Omega \subset \mathbb{R}^N$, $N \geq 2$, be a bounded domain with Lipschitz boundary. This paper studies a nonlinear transmission eigenvalue problem driven by nonhomogeneous operators with different p-growth in two subdomains Ω_1 and Ω_2, separated by an interface Σ. The model incorporates continuity and flux transmission conditions across the interface, while the eigenvalue parameter λ appears both in the differential equations and in the nonlinear boundary conditions.

The main result establishes the existence of an unbounded sequence of eigenvalues for the transmission problem. The proof combines the Krasnosel'skii genus with a Lusternik–Schnirelmann type minimax principle on a suitable C^1-manifold and exploits compactness and coercivity properties of the associated energy functional. As a consequence, infinitely many eigenpairs are obtained and the corresponding eigenvalues tend to infinity.

The obtained result extends several known eigenvalue existence results for nonlinear elliptic equations to a transmission setting involving nonhomogeneous operators

***K*eywords** Nonlinear transmission eigenvalue problem, nonhomogeneous operators with p-growth, Krasnosel'skii genus

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ON THE SPECTRUM OF A NONLINEAR TRANSMISSION PROBLEM WITH DIFFERENT GROWTH CONDITIONS

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ABSTRACT

This work focuses on the spectral analysis of a nonlinear transmission problem involving nonhomogeneous operators acting in adjacent subdomains $\Omega_i,\ i = \overline{1,2}$ and coupled through interface conditions on Σ. There are different growth rates $p_i,\ i = \overline{1,2}$ of the operators in each region and we also have boundary conditions on $\Gamma_i,\ i = \overline{1,2}$.

The main result establishes that, under suitable assumptions, every positive real number λ is an eigenvalue of the transmission problem. The proof relies on variational techniques and has two different growth regimes.

In the superlinear case, suitable estimates are used to obtain compactness properties of the associated energy functional, which allow the application of the Mountain Pass Theorem and guarantee the existence of nontrivial critical points.

In the sublinear case, the existence of eigenfunctions follows from the coercivity and a minimization method.

***K*eywords** Mountain Pass Theorem · variational methods · nonlinear transmission problem

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A SPECIAL CASE OF $\oplus$-G-SUPPLEMENTED MODULES

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ABSTRACT

In this work, every ring has an identity and every module over a ring R is a unitary left R-module. Let M be an R-module and $U, V \leq M$. If $M = U + V$ and V is minimal for this property, then V is called a supplement of U in M. V is a supplement of U in M if and only if $M = U + V$ and $U \cap V \ll V$. If every submodule of M has a supplement in M, then M is called a supplemented module. If every submodule of M has a supplement that is a direct summand in M, then M is called a $\oplus$-supplemented module. Let M be a supplemented module. If every supplement submodule in M is a direct summand of M, then M is called a strongly $\oplus$-supplemented module. Let M be an R-module and $N \leq M$. If $T = M$ for every $T \trianglelefteq M$ with $M = N + T$, then N is called a g-small (or g-superfluous) submodule of M and denoted by $N \ll_g M$. Let M be an R-module and $U, V \leq M$. If $M = U + V$ and $U \cap V \ll_g V$, then V is called a g-supplement of U in M. If every submodule of M has a g-supplement in M, then M is called a g-supplemented module. If every submodule of M has a g-supplement that is a direct summand in M, then M is called a $\oplus$-g-supplemented module. Let M be a g-supplemented module. If every g-supplement submodule in M is a direct summand of M, then M is called a strongly $\oplus$-g-supplemented module. In this work, some new properties of strongly $\oplus$-g-supplemented modules are investigated. It is clear that every strongly $\oplus$-g-supplemented module is $\oplus$-g-supplemented. But the converse of this statement is not true in general.

Keywords Modules · g-Small Submodules · Supplemented Modules · g-Supplemented Modules

2020 Mathematics Subject Classification: 13C13, 13C60.

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RADICAL INJECTIVE MODULES

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ABSTRACT

In this work, every ring has an identity and every module over a ring R is a unitary left R-module. Let M be an R-module and $N \leq M$. If there exists a submodule K of M such that $M = N + K$ and $N \cap K = 0$, then N is called a direct summand of M and it is denoted by $M = N \oplus K$. For any R-module M, we have $M = M \oplus 0$. Let M be an R-module. Let M be an R-module and $N \leq M$. If $L = M$ for every submodule L of M such that $M = N + L$, then N is called a small (or superfluous) submodule of M and denoted by $N \ll M$. A proper submodule N of an R-module M is called a maximal submodule of M if there exist no proper submodules with $N \leq K \leq M$ and $K \neq N$. The radical of a module M is defined the intersection of all maximal submodules of M and denoted by $RadM$. If M have no maximal submodules then the radical of M is defined by $RadM = M$. Let M and N be two R-modules. M is said to be N-injective if every diagram

$$\begin{array}{ccccc} 0 & \longrightarrow & K & \xrightarrow{g} & N \\ & & \downarrow f & \swarrow & \\ & & M & & \end{array}$$

with exact row can be extended commutatively by an R-module homomorphism $N \longrightarrow M$. If M is T-injective for every R-module T, then M is called an injective R-module. Let M and N be two R-modules. N is said to be radical M-injective (briefly, r-M-injective) if for every exact sequence $0 \longrightarrow K \xrightarrow{g} M$ with $RadM \leq g(K)$ and every R-module homomorphism $f : K \longrightarrow N$, there exists an R-module homomorphism $h : M \longrightarrow N$ with $h \circ g = f$. If N is radical T-injective for every R-module T, then N is called a radical injective (briefly, r-injective) module. In this work, some properties of radical injective modules are investigated.

***K*eywords** Modules · Small Submodules · Radical · Injective Modules

2020 Mathematics Subject Classification: 16D40, 16D50.

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GENERATING RIORDAN MATRICES VIA HIGHER-ORDER RECURRENCES

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ABSTRACT

The Riordan group, denoted by R, is a set of infinite lower-triangular matrices defined by a pair of formal power series $(g(t), f(t))$ such that $g(0) \neq 0, f(0) = 0$, and $f^{'}(0) \neq 0$. The generic element $T = (d_{n,k})_{n,k\geq 0}$ of the matrix is given by the property that the generating function of its $k-$th column is $g(t)(f(t))^k$. In this study, we establish explicit Riordan representations for a class of Riordan matrices generated by number sequences satisfying higher-order recurrence relations. Utilizing these algebraic representations, we investigate the fundamental combinatorial properties inherent in the structure of these matrices. Furthermore, we examine various row sum characteristics and derive explicit algebraic formulas for specific row sums.

***K*eywords** Riordan group, · Formal power series · Higher-order recurrence relation, · Combinatorial identities, · Row sums

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BEYOND CONTENT REMEDIATION: REDESIGNING A FIRST-YEAR ENGINEERING MATHEMATICS WORKSHOP

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ABSTRACT

Mathematics bridging courses are a widely used strategy to support students' transition into engineering programmes. Traditionally, these initiatives have prioritised the reinforcement of mathematical content, whereas recent frameworks for engineering mathematics education propose broadening this focus towards the development of mathematical competencies that enable students to understand, use and communicate mathematics in disciplinary and professional contexts ([3]; [5]). Against this background, this paper describes the redesign process of the Logical and Mathematical Aptitudes Workshop (TALM), a compulsory course within the engineering curriculum of a Chilean university, aimed at first-year students, whose initial instructional design, grounded in the diagnosis of incoming students' mathematical gaps, was previously presented by Arce et al. ([1]). Following its implementation, the course evolved from a content-centred remedial model towards an approach oriented towards the development of transversal mathematical competencies. The redesign was grounded in the experience accumulated during the 2024 and 2025 implementations, the analysis of student perceptions collected during the 2025 implementation, and the review of theoretical frameworks on mathematical competencies, engineering education, constructive alignment and active learning ([2]; [3]; [5]; [6]). The evidence collected, together with the review of the specialised literature, informed a process of reconceptualising the course's formative purpose, strengthening the explicit development of mathematical competencies and the coherence between learning outcomes, teaching activities and assessment processes. As a result, TALM was reformulated around four transversal mathematical competencies: mathematical reasoning, mathematical modelling, problem solving and mathematical communication. The new design organises learning through active methodologies and activities contextualised in engineering-related situations, fostering connections between mathematics and students' future professional practice. It also incorporates an assessment system aligned with the learning outcomes, complemented by pre- and post-test instruments to monitor competency development, and integrates the TALM Checklist, a didactic tool inspired by the Decoding the Disciplines approach ([4]). Its purpose is to make explicit the cognitive operations involved in mathematical problem solving, guiding students through the processes of modelling, analysis and validation of their answers. It is proposed that this redesign offers an alternative way of rethinking bridging courses in engineering, shifting the focus from the remediation of prior content towards the development of transferable mathematical competencies through contextualised learning experiences, assessment coherent with learning outcomes, and strategies that foster the explicit articulation of disciplinary mathematical thinking.

Keywords engineering mathematics education · student transition to engineering · transversal mathematical competencies · competency-based redesig · contextualised learning · active learning · constructive alignment.

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ASYMPTOTIC MEIR KEELER CONTRACTION MAPPINGS ON MULTIPLICATIVE METRIC SPACES WITH AN INTEGRAL EQUATION APPLICATION

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ABSTRACT

In current work, multiplicative nonlinear contraction mapping, multiplicative Meir Keeler contraction mapping, multiplicative asymptotic contraction mapping and multiplicative asymptotic Meir Keeler contraction mapping are defined and investigated on multiplicative metric spaces. After that, considering these mappings, the new fixed point theorems are obtained and proved in multiplicative metric spaces. Finally, an interesting application related to integral equation is presented to make the subject clearer and more understandable from a different perspective. .

***K*eywords** Multiplicative Metric · Multiplicative Mapping · Asymptotic Contraction · Meir Keeler Contraction · Asymptotic Meir Keeler Contraction · Fixed Point

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COEFFICIENT INEQUALITIES FOR A SUBCLASS OF BI-UNIVALENT FUNCTIONS INVOLVING GEGENBAUER POLYNOMIALS

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ABSTRACT

In this paper, we establish bounds for the initial Taylor–Maclaurin coefficients $|a_2|$ and $|a_3|$ for a new subclass of analytic and bi-univalent functions in the open unit disk involving Gegenbauer polynomials. Furthermore, we investigate several special cases and consequences of this new subclass.

***K*eywords** Analytic and bi-univalent functions · Subordination · Coefficient inequalities · Gegenbauer polynomials

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FEKETE-SZEGÖ INEQUALITIES FOR A SUBCLASS OF BI-UNIVALENT FUNCTIONS INVOLVING GEGENBAUER POLYNOMIALS

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ABSTRACT

In this paper, we obtain bounds for the Fekete–Szegö inequalities for a new subclass of analytic and bi-univalent functions in the open unit disk involving Gegenbauer polynomials. Furthermore, we investigate several special cases and consequences of this new subclass

***K*eywords** Analytic and bi-univalent functions · Subordination · Fekete–Szegö inequality · Gegenbauer polynomials

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INITIAL COEFFICIENT INEQUALITIES FOR A SUBCLASS OF BI-UNIVALENT FUNCTIONS INVOLVING MERSENNE POLYNOMIALS

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ABSTRACT

In this article, we derive bounds for the initial Taylor–Maclaurin coefficients $|a_2|$ and $|a_3|$ for a newly defined subclass of analytic and bi-univalent functions in the open unit disk associated with Mersenne polynomials. In addition, several special cases and consequences of the proposed subclass are discussed.

***K*eywords** Analytic and bi-univalent functions · Subordination · Coefficient inequalities · Mersenne polynomials

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ON FEKETE-SZEGÖ INEQUALITIES FOR A SUBCLASS OF BI-UNIVALENT FUNCTIONS INVOLVING MERSENNE POLYNOMIALS

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ABSTRACT

In this paper, we obtain bounds for the Fekete–Szegö inequalities for a new subclass of analytic and bi-univalent functions in the open unit disk involving Mersenne polynomials. Furthermore, we investigate several special cases and consequences of this new subclass

***K*eywords** Analytic and bi-univalent functions · Subordination · Fekete–Szegö inequality · Mersenne polynomials

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AN INTEGRATED CLUSTERING - FUZZY TOPSIS - MILP FRAMEWORK FOR INDUSTRIAL FACILITY LOCATION OPTIMIZATION: THE CASE OF A DAP FERTILIZER PLANT IN SENEGAL

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ABSTRACT

Senegal possesses one of the world's largest phosphate rock reserves, yet its diammonium phosphate (DAP) production capacity remains limited, creating a strategic opportunity for domestic value-added fertilizer manufacturing. Selecting an optimal site for such an industrial facility is a multidimensional problem that must reconcile qualitative locational criteria with quantitative economic viability. This research is situated within the broader context of the Facility Location Problem, drawing upon solution frameworks from both Operations Research and Multi-Criteria Decision-Making. Our study proposes a hybrid computational decision-support framework integrating four complementary methods. First, a Geographic Information System (QGIS) maps candidate zones as phosphate deposits and defines a study area based on a metric coordinate system. Second, K-Means clustering groups these candidate zones into homogeneous spatial clusters, reducing the problem to a tractable set of alternatives. Third, a Fuzzy TOPSIS multi-criteria decision-making model built on triangular fuzzy numbers, ranks the resulting clusters according to a composite of economic, logistical, and environmental criteria. Fourth, a Mixed-Integer Linear Programming (MILP) model, calibrated with West African market parameters (DAP price, production capacity, capital recovery factor, operating costs etc.), independently evaluates each cluster's expected net profitability. Applying this framework, Cluster 0 emerges as the optimal location under both methods, with a Fuzzy TOPSIS closeness coefficient of 0.6199 and a projected MILP net profit of approximately USD 42.88 million per year, yielding a 60% ranking concordance between the two approaches. To confirm the robustness of this study across various scenarios and to standardize this model, a multimodal sensitivity analysis of the key parameters is recommended. The proposed framework offers a replicable, mathematically grounded decision-support tool for industrial facility location problems, with direct relevance to resource-based industrialization strategies in developing economies.

Keywords Facility location · Fuzzy TOPSIS · Mixed-Integer Linear Programming · K-Means clustering · GIS · DAP fertilizer · Senegal

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IMPACT OF FEAR ON PREY IN A LESLIE-GOWER TYPE MODEL WITH GENERALIST PREDATORS

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ABSTRACT

Different ecological phenomena have been included in the study of the interaction between a prey and its principal predatory population.

Facts as refuge use by prey, or actions as the formation of group of defence are some of these disturbances in the important interrelation.

In this paper, a Leslie-Gower-type predation model assuming that populations act with fear of facing predators is analyzed, which is described by a nonlinear ordinary differential equation system (ODEs).

The model's dynamical properties including boundedness, positivity, extinction criteria, and the existence and stability (both local and global) of equilibrium points are rigorously examined.

We will consider how the collective fear of prey towards their predators affects the growth rate of the prey. Furthermore, his ecological phenomenon can be mathematically described by different functions. For example, the function $f(\alpha, \eta, y) = \eta$, express the fear, showing that if the predator population increases in the environmental, the prey population experiences minimum fear pressure from the predator species.

One of the purposes of this work is to point out that some of these new systems with such modifications can have the same topological portrait in the phase plane as the original one.).

***K*eywords** Predator-prey, bifurcation, stability

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NEW FIXED POINT THEOREMS ON DIGITAL RECTANGULAR METRIC SPACE AND APPLICATION OF LACTIC ACID FERMANTATION

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ABSTRACT

In this study we consider digital images with adjacency relation. We provided an example that is a digital rectangular metric but not a digital metric. Moreover, we examined the example of lactic acid fermentation in a digital rectangular metric space and define a contraction mapping that preserves homotopy equivalence. Furthermore, we prove a Chatterjea-type fixed point theorem in a digital rectangular metric space and demonstrate that the contraction mapping which we define within the digital topological structure of lactic acid fermentation satisfies the Chatterjea-type contraction condition.

***K*eywords** Digital image · Rectangular metric · Fixed point

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THE ISOPERIMETRIC PROBLEM FOR HYPERIDEAL POLYHEDRA

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ABSTRACT

The isoperimetric problem for polyhedra originates from a question posed by Jakob Steiner in 1842: within its combinatorial class and for a fixed surface area, does a regular Euclidean polyhedron maximize volume? This question has been answered affirmatively for the regular tetrahedron, the regular octahedron, the cube, and the regular dodecahedron.

A point in the hyperbolic space $\mathbb{H}^3$ is classified as type 1, 0, or -1 according to whether it lies inside $\mathbb{H}^3$, on the sphere at infinity, or beyond the sphere at infinity, respectively. A *hyperideal polyhedron* is a polyhedron whose vertices are all of type 0 or -1. The isoperimetric problem can likewise be formulated for hyperideal polyhedra. Frigerio and Moraschini [2] solved the isoperimetric problem for truncated hyperideal tetrahedra.

In this paper, we study hyperideal polyhedra whose vertices are all of type -1, with every vertex truncated. We establish the following result:

A convex hyperideal polyhedron P_0 with all vertices of type -1 and equal edge lengths uniquely maximizes the volume among all convex hyperideal polyhedra with vertices of type -1 sharing the same combinatorial type and surface area as P_0.

Its proof relies on two fundamental properties of hyperideal polyhedra: the convexity of the space of dihedral angles, established by Bao and Bonahon [1], and the strict concavity of the volume function, established by Schlenker [3, 4].

As a consequence of this theorem, we obtain infinitely many hyperideal polyhedra that solve the isoperimetric problem within their combinatorial classes. The first family of such examples consists of hyperideal polyhedra with equal edge lengths constructed from Euclidean convex polyhedra whose faces are regular polygons.

The second family consists of hyperideal polyhedra with equal edge lengths that have no Euclidean counterparts.

Keywords isoperimetric problem · hyperideal polyhedra · hyperbolic volume · convex uniform polyhedra · Johnson solids

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NEW PAWLAK APPROXIMATIONS USING IDEAL WITH ECONOMIC APPLICATIONS

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ABSTRACT

Rough set theory is essentially concerned with equivalence relations. The theory, introduced by Pawlak, is based on Pawlak Approximations—consisting of the lower and upper approximations of any subset within the given Pawlak Approximation Space. The abstactions of rough sets which regarding topological structures have become important area in rough set theory. In this paper associated with j-neighborhood space and ideal, a new method for generalizing Pawlak rough sets are presented and their properties are investigated. It is compare with some of the former methods. Furthermore, an economic application is given as an example to show the idea suggested.

***K*eywords** rough sets · j-neighborhood spaces · ideal · topology

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ROUGH SET MODELS CAUSED BY NANO TOPOLOGIES ON FINITE DIRECTED GRAPHS

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ABSTRACT

Nano topology is matcless topology using rough set theory. Really, it is consist of maximum of five elements and introduced by Thivagar and Richard. On the other hands, graph theory is a popular subject in science such as chemistry as well as mathematics. It is known that graph theory and topology are related two subject. In this paper states a new topology originate from graphs and at same time applies it to obtain an original rough set model. The essential properties of this model is investigated and compare it with previous ones. The result indicates that this approach is better than earlier method.

***K*eywords** rough sets · topology · nano topology · graphs

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A New Class of Neural Network Operators for Approximation Theory

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Abstract

Neural network operators have become an interesting topic in approximation theory because they provide a different approach to approximating continuous functions. Their construction is closely related to the activation function and the choice of suitable weights. In this paper, we introduce a new class of neural network operators and study their approximation properties. The proposed operators are defined by using a suitable activation function together with a sequence of weights. We first examine the basic properties of the operators and give conditions under which they are well defined, positive and bounded. We then study their convergence to the function being approximated. The convergence is considered in an appropriate function space and is described in terms of the usual tools of approximation theory. In addition, we investigate the rate of approximation of the proposed operators by using the modulus of continuity. This gives an estimate of the error between a function and its corresponding approximation. The effect of the parameters involved in the operators is also discussed, since these parameters have an important role in the quality of approximation. Under suitable choices of the activation function and the parameters, the proposed operators include some known neural network-type approximation processes as special cases. The results obtained in this work provide a mathematical framework for studying neural network operators from the point of view of classical approximation theory. They may also be useful for further investigations of generalized neural network operators and their applications to the approximation of continuous and nonlinear functions.

Keywords Approximation theory · neural network operators · activation functions · pos- itive operators · convergence · modulus of continuity · rate of approximation · continuous functions

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A Robust Hybrid Semi-Analytical Analysis of Time-Fractional Black–Scholes European Option Pricing Model Emerging in Financial Markets

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Abstract

Fractional differential equations generalize the classical concept of integer-order derivatives, enabling the incorporation of memory effects and historical states into mathematical models. The ability of fractional-order models to more accurately represent time-dependent interactions and long-range dependencies observed in real-world systems has attracted considerable attention in recent years. These equations provide valuable tools for the mathematical analysis of phenomena that cannot be adequately described by classical differential equations, while offering greater flexibility in modeling complex systems [1]. One of the prominent applications of this approach is found in fractional option pricing models developed within the field of financial mathematics. Classical option pricing theory is fundamentally based on the assumptions of arbitrage-free markets and dynamic hedging strategies; however, it has been extended by various researchers through the introduction of fractional derivative concepts. In this context, the fractional Black–Scholes model for European option pricing allows for a more realistic evaluation of option values by incorporating memory effects and irregular price dynamics observed in financial markets. Such models have found widespread applications in risk management, derivative pricing, financial engineering, and investment strategy development, and continue to constitute an important area of research in modern financial theory [2, 3]. Analytical and semi-analytical techniques play a crucial role in solving fractional differential equations. Among these, integral transform methods have proven to be effective tools for reformulating differential problems into more tractable mathematical forms. The Kashuri Fundo Transform, introduced in recent years, together with the Homotopy Perturbation Method, which is widely employed for nonlinear problems, has been recognized as a powerful framework for obtaining highly accurate approximate solutions [4]. In this study, the applicability of these complementary techniques is investigated for a fractional-order financial model. The obtained results demonstrate that the proposed approach simplifies the solution procedure and facilitates the construction of convergent solutions. Therefore, it may provide an alternative and effective solution framework for fractional models encountered in financial mathematics.

***K*eywords** Black-Scholes European option pricing · Kashuri Fundo transform · homotopy perturbation method · fractional Black-Scholes model

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A New Integral Transform-Based Numerical Analysis of Fractional Gas Dynamics Equation

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Abstract

Gas dynamics equations arise as mathematical representations of the fundamental conservation laws of mass, momentum, and energy in physical systems. Gas dynamics, a significant branch of fluid dynamics, investigates the motion of gases and the effects of this motion on physical systems [1, 2]. Studies in gas dynamics contribute significantly to the understanding of many critical physical phenomena, such as the formation of shock waves around aircraft and aerodynamic heating occurring on vehicle surfaces during atmospheric re-entry. Therefore, gas dynamics equations occupy an important position in both theoretical and applied sciences. Fractional gas dynamics equations represent a generalized form of classical gas dynamics models and have been developed to more realistically describe complex physical phenomena, particularly memory effects and anomalous diffusion [1, 2]. With the introduction of fractional derivative concepts into physical systems, these equations have gained considerable attention in the literature and have become an active area of research in engineering and physics. In recent years, various semi-analytical methods have been proposed to effectively handle the complex structure of these equations. In this context, hybrid approaches formed by combining integral transform techniques with analytical or algebraic methods have emerged as powerful tools for solving fractional differential equations. These methods have found widespread application in the literature due to their computational efficiency and their ability to generate rapidly convergent series solutions. In this study, the Kashuri–Fundo Decomposition Method (KFDM) is employed to solve fractional gas dynamics equations [3, 4]. The method is based on the decomposition of nonlinear terms through Adomian polynomials, enabling the solution to be obtained in the form of a rapidly convergent series. The results demonstrate that KFDM can produce highly accurate solutions of the series expansion and yields results that are consistent with those reported by other semi-analytical methods in the existing literature. These findings indicate that KFDM provides an effective and reliable alternative approach for solving fractional differential equations.

***K*eywords** Fractional gas dynamics equation · Kashuri Fundo decomposition method · fractional differential equation · Adomian decomposition method

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A Permanent Representation of Perrin Numbers via a Certain Complex Pentadiagonal Matrix

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Abstract

Permanents play an important role in various branches of mathematics and applied sciences, including graph theory, combinatorics, physics, chemistry, electrical engineering, and computational mathematics. In particular, establishing relationships between permanents of structured matrices and well-known integer sequences has attracted considerable interest, since such representations provide elegant matrix interpretations together with effective computational techniques for recursive sequences [1, 2]. In recent years, several studies have investigated determinant and permanent representations of Fibonacci-, Pell-, and Perrin-type sequences by means of Hessenberg and related structured matrices, revealing remarkable connections between matrix theory and number theory [3, 4, 5].

Among third-order linear recurrence sequences, the Perrin sequence possesses remarkable algebraic, combinatorial, and number-theoretic properties and has applications in recurrence theory and primality testing [6]. The Perrin sequence $\{R_n\}$ is defined by

$$R_n = R_{n-2} + R_{n-3}, \qquad n \geq 3,$$

with initial conditions $R_0 = 3$, $R_1 = 0$, and $R_2 = 2$.

Motivated by previous matrix representations of Perrin numbers, this paper introduces a new class of complex pentadiagonal matrices and investigates their permanents. We prove that the permanent of the proposed complex pentadiagonal matrix coincides with the corresponding Perrin number, thereby establishing a new permanent representation of the Perrin sequence. The proof is based on the recursive expansion formula for permanents together with the special structure of the proposed matrix. Furthermore, a Maple procedure is developed to verify the theoretical results and to compute the permanents efficiently for matrices of arbitrary order.

The proposed construction extends the existing literature by introducing a complex pentadiagonal matrix representation for Perrin numbers, whereas previous studies have mainly focused on Hessenberg matrices. Consequently, the obtained results provide an alternative and computationally efficient framework for studying Perrin numbers and further strengthen the relationship between permanents of structured matrices and third-order linear recurrence sequences.

***K*eywords** Perrin numbers · permanent · pentadiagonal matrix · integer sequences

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AN EXPLICIT EIGENVALUE DECOMPOSITION OF CIRCULANT TRIDIAGONAL MATRICES

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ABSTRACT

Circulant matrices constitute one of the most extensively studied classes of toeplitz matrices because of their remarkable algebraic properties and their wide range of applications in signal processing, image restoration, coding theory, numerical linear algebra, and the numerical solution of differential and integral equations. A comprehensive treatment of the theory of circulant matrices was given by Davis [1], where their algebraic structure, polynomial representation, relationship with the discrete Fourier transform, and numerous applications were systematically presented. In particular, Davis showed that every circulant matrix can be diagonalized by the discrete Fourier matrix, establishing the foundation for subsequent studies on their spectral properties.

Based on the Fourier representation introduced by Davis, Karner et al. [2] investigated the spectral decomposition of several classes of real circulant matrices, including right circulant, left circulant, skew right circulant, and skew left circulant matrices. They explicitly constructed the corresponding eigendecompositions and singular value decompositions, revealing the relationship between circulant matrices, the discrete Fourier transform, and orthogonal transformations.

Later, Köken and Bozkurt [3] considered a special class of odd-order circulant tridiagonal matrices and obtained explicit formulas for their positive integer powers. Their approach was based on the eigenvalue decomposition of circulant matrices together with trigonometric identities involving Chebyshev polynomials. Subsequently, Öteleş and Akbulak [4] generalized these results by deriving a unified closed-form expression valid for both odd- and even-order circulant tridiagonal matrices. Their formulation considerably simplified the computation of matrix powers and demonstrated the effectiveness of Chebyshev polynomial techniques in the analysis of structured matrices.

More recently, Aliatiningtyas et al. [5] revisited the spectral theory of general circulant matrices and presented explicit formulas for their eigenvalues, determinants, inverses, and invertibility conditions. Their work also emphasized that the eigenvalues of circulant matrices can be efficiently computed through the discrete Fourier transform, providing a computationally attractive framework for spectral analysis.

Motivated by these studies, this paper considers the real circulant tridiagonal matrix

$$A_n = \operatorname{circ}(a_0, a_1, 0, \ldots, 0, a_{-1}).$$

By using the classical Fourier representation, we derive an explicit eigenvalue decomposition of A_n. The resulting eigendecomposition provides an efficient framework for computing integer powers, inverse matrices, matrix functions, and other spectral quantities. Consequently, the obtained results extend several existing studies on circulant tridiagonal matrices and provide a useful tool for both theoretical investigations and computational applications.

Keywords circulant matrix · eigenvalue decomposition · Fourier matrix

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COMPUTABLE EXPONENTIAL-TYPE BOUNDS FOR THE EXPECTED RESIDUAL WAITING TIME UNDER HEAVY-TAILED DISTRIBUTIONS

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ABSTRACT

The residual waiting time process $W(t)$ arises in many application areas, including queueing systems, reliability analysis, stock-control and inventory systems. However, except for certain special distributions such as the Erlang distribution, obtaining closed-form expressions for the expected residual waiting time $\mathbb{E}[W(t)]$ is generally analytically difficult. The aim of this study is to obtain valid exponential-type bounds for the expected residual waiting time $\mathbb{E}[W(t)]$. Since $\mathbb{E}[W(t)]$ and the renewal function $U(t)$ are related by $\mathbb{E}[W(t)] = \mu U(t) - t,\ \mu = \mathbb{E}[X] < \infty$, the problem can be reduced to deriving lower and upper bounds for $U(t)$. The resulting bounds obtained in this study remain valid, including heavy-tailed cases for which $\mathrm{Var}(X) = \infty$.

The $\kappa(t)$-based exponential-type bounds proposed by Chadjiconstantinidis (2024) provide sharp results for $U(t)$. However, since $\kappa(t)$ is defined by the integral root equation

$$\int_0^t e^{\kappa(t)y}\, dF(y) = 1,$$

its evaluation generally requires numerical computation in practice.

Building upon the approach of Chadjiconstantinidis (2024), this study first derives a lower bound $\kappa_{\mathrm{low}}(t)$ by using the convexity of the exponential function. An upper bound $\kappa_{\mathrm{up}}(t)$ is then obtained with the aid of the truncated second moment $m_2(t) = \mathbb{E}\left[X^2 \mathbf{1}_{\{X \le t\}}\right]$. Thus, the analytic bracketing $\kappa_{\mathrm{low}}(t) \le \kappa(t) \le \kappa_{\mathrm{up}}(t)$ is established. This interval is subsequently narrowed by the bisection method, yielding bounds for $U(t)$ whose validity is preserved, and consequently lower and upper bounds for $\mathbb{E}[W(t)]$.

The numerical results demonstrate that the $\kappa(t)$-based values reported in the literature can be approached with only a small number of iterations, for the heavy-tailed numerical examples considered in this study, while the bounding property is preserved at every step.

Keywords Residual waiting time · Renewal function · Heavy-tailed distributions · Exponential-type bounds · Bisection method

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SOME NORMAL APPROXIMATIONS FOR THE VARIANCE OF A RESIDUAL WAITING TIME PROCESS

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ABSTRACT

The residual waiting time process is a fundamental quantity in renewal theory and represents the time remaining from an arbitrary observation epoch until the next renewal event. Its mean and variance are important performance measures in queueing, reliability, and inventory systems; however, they are generally difficult to evaluate because the associated renewal function rarely admits a closed-form expression. This study develops computationally tractable normal approximations for the mean and variance of the residual waiting time process when the interarrival times follow a Weibull distribution with shape parameter $\beta \geq 3$. In this parameter range, the Weibull distribution becomes less skewed and can be effectively approximated by a normal distribution with matching first two moments.

Let $U(t)$ denote the renewal function and $W(t)$ the residual waiting time. By Wald's identity, $\mathbb{E}[W(t)] = \mu U(t) - t$, where μ is the mean interarrival time. Using a truncated normal series representation for $U(t)$ developed by Cui and Xie (2003), we provided three approximations for $E(W(t))$. Their accuracy is assessed through Monte Carlo simulation for Weibull interarrival times. The numerical results show close agreement with the simulated values.

The analysis is further extended to the variance of $W(t)$. For validation, an explicit variance formula is derived for Erlang$(3, \lambda)$ interarrival times, and the case $\lambda = 1$ is examined numerically using Simpson's rule for the integral term. The variance approximations become highly accurate as t increases, with relative errors below 1% for $t \geq 6$, and converge to the equilibrium variance $8/3$. The results demonstrate that the proposed method provides a simple, efficient, and accurate alternative for evaluating the first two moments of residual waiting time when direct renewal calculations are unavailable.

Keywords Residual waiting time · Renewal function · Normal approximation · Weibull distribution · Erlang distribution

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THE WAVELET TRANSFORM RELATED TO THE JACOBI–CHEREDNIK OPERATOR AND ITS APPLICATIONS

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ABSTRACT

In this paper, we introduce the wavelet transform associated with the Jacobi–Cherednik operator. We first present some basic properties of the Opdam–Cherednik wavelet transform. Then, we establish a few versions of the uncertainty principle for this transform. Also, we show that the Opdam–Cherednik convolution operator is a time-invariant filter and give a physical interpretation of a time-invariant filter. As an application, we discuss a signal recovery problem using uncertainty principles and show that the reconstruction of a transmitted signal from a noisy received signal is possible using the uncertainty principle for the Opdam–Cherednik wavelet transform. Finally, we solve the Fredholm-type integral equation using the time-invariant filter for this transform.

***K*eywords** Opdam–Cherednik wavelet transform · Uncertainty principles · Signal recovery · Time-invariant filter · Fredholm integral equation

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AN EXTENSION OF MARKOV-TYPE OPERATORS. APPLICATIONS TO THE FRACTAL THEORY.

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ABSTRACT

In this paper, we provide an extension of the concept of Markov-type operator. Classical Markov-type operators associated with iterated function systems have traditionally been studied either for scalar-valued measures or for vector measures taking values in a Hilbert space, a restriction that leaves out a substantial class of infinite-dimensional settings in which such operators arise naturally. Motivated by this gap, we consider the case when the Markov-type operator, associated to a family of contractions together with a compatible family of bounded linear operators, acts on Borel measures of bounded variation which take values in the conjugate of an arbitrary Banach space. This generalization is carried out uniformly for finite, countable, and uncountable families of contractions, thereby unifying constructions that are usually treated separately in the existing literature. We give the main properties of this operator, including its boundedness on the space of Banach dual-valued measures, explicit operator norm estimates in terms of the norms of the associated linear operators and the contraction ratios, and change-of-variable formulas relating the operator to vector-valued integration. These results are obtained with respect to a Monge-Kantorovich type norm defined on the space of vector measures, which provides the appropriate functional framework for studying continuity and convergence properties of the operator. Then, using the Markov-type operator, we construct a non-linear operator, which has, in some cases, a unique fixed point, named the fractal measure. Sufficient contractivity conditions, expressed through the operator norms and the contraction ratios of the underlying maps, are established under which the Banach Fixed Point Theorem guarantees the existence and uniqueness of this measure. One can find a connection between the fractal measure and the attractor, that is, the fractal set, associated to the family of contractions, showing that the support of the fractal measure coincides with this attractor in appropriate settings and thereby preserving the classical geometric interpretation inherited from the Hutchinson construction. We also give some examples, in both finite-dimensional and infinite-dimensional Banach spaces, to illustrate the theoretical facts and to demonstrate that the derived contractivity conditions can be verified explicitly in concrete situations.

***K*eywords** Markov operator · attractor · fractal measure · iterated function system · Monge-Kantorovich norm

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FROM ANSWER GENERATION TO CRITICAL VALIDATION: AN AI-MEDIATED ASSESSMENT DESIGN IN ENGINEERING MATHEMATICS

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ABSTRACT

Generative artificial intelligence has made fluent solutions to mathematical problems instantly available to students, shifting the central question in engineering mathematics from whether AI is used to how assessment can make disciplinary judgement visible. This paper reports a classroom-based study of an assessment design in which AI output is deliberately externalised and converted into an object of mathematical validation. The design was implemented in two courses at the same institution — an undergraduate Statistical Methods course in Mechanical Engineering (74 students) and a master's-level Applied Mathematics to Engineering course (34 students) — through three tasks per cycle. In each task, students first produced their own solution, then consulted a generative system, documented prompts and outputs, compared the two lines of reasoning, and justified a final judgement. Work was assessed through a common rubric with three dimensions: Rigour, AI Analysis, and Clarity. Because the tasks contributed to summative marks, associations with achievement were computed against a task-independent component of the final grade. Within the master's cohort, all three dimensions improved significantly across the task sequence (Friedman, $p < .005$; for AI Analysis, T1 to T3 Cliff's $\delta = 0.56$). The undergraduate cohort showed no comparable progression: Rigour remained stable, AI Analysis fell in the second task before partially recovering, and Clarity declined monotonically with a large effect ($\delta = -0.74$, $p < .001$). Undergraduate participation also fell from 74 students in the first task to 35 in the third, with no evidence that those who withdrew were the weaker performers. The master's cohort scored higher throughout, and the gap widened across tasks. Read through the RBC+C model of abstraction in context (Hershkowitz, Schwarz, & Dreyfus, 2001), the divergence is one of consolidation, which is theorised to require repeated opportunities to revisit and reorganise prior constructs. Cohort maturity and prior selection plausibly account for the difference in level, but not for the divergence in direction: one cohort consolidated while the other deteriorated. The evidence is more consistent with cumulative workload in a large undergraduate class, an interpretation supported by time management being the lowest-rated item in both cycles and by the observed attrition. The paper argues that the educational value of generative AI lies in assessment designs that convert AI output into an object of validation and critique, and that the same design does not transfer across cycles without attention to task load, cohort size, and the number of documented AI interactions students can sustain.

***K*eywords** generative AI · engineering mathematics · assessment redesign · evaluative judgement · AI literacy · mathematics education.

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IDENTITIES ON TRIANGULAR FUZZY PELL AND TRIANGULAR FUZZY PELL–LUCAS NUMBERS AND A FUZZY TOPSIS APPLICATION

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ABSTRACT

Classical Pell and Pell-Lucas number sequences are integer sequences that have been extensively studied and have a wide range of applications in number theory and applied mathematics. In this study, these sequences are transferred to the fuzzy setting as triangular fuzzy Pell and triangular fuzzy Pell-Lucas numbers, defined through the α-cut representation of triangular fuzzy numbers. First, a basic arithmetic framework for triangular fuzzy numbers is established, and this framework is used to derive the recurrence relations of the proposed fuzzy sequences. Building on this foundation, several new identities are obtained that reveal the relationships between triangular fuzzy Pell and triangular fuzzy Pell-Lucas numbers; in this way, classical results are extended to the fuzzy domain, and the algebraic relationships between the two families of sequences are clarified.

As an application, the proposed triangular fuzzy Pell numbers are incorporated into a multi-criteria decision-making framework based on the Fuzzy TOPSIS method. The linguistic variables used in the decision-making process are modeled by triangular fuzzy numbers derived from fuzzy Pell numbers, and the resulting ranking is compared with a parallel solution obtained using classical Pell numbers. The two approaches yield consistent rankings, which supports the practical reliability of the fuzzy Pell-based formulation.

The results show that triangular fuzzy Pell and triangular fuzzy Pell-Lucas numbers form a consistent algebraic structure compatible with existing fuzzy number theory, while also providing a concrete and effective tool for multi-criteria decision-making applications. This study contributes to the growing literature at the intersection of number sequences and fuzzy set theory, and suggests future research directions such as extensions to other classical sequences and alternative fuzzy decision-making methods.

***K*eywords** Triangular fuzzy Pell numbers · Triangular fuzzy Pell-Lucas numbers · α -cut · Fuzzy TOPSIS

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SEQUENTIAL HYPOTHESIS TESTING UNDER CONTINUOUS MONITORING: SEQUENTIAL E-VALUES AND SEQUENTIAL P-VALUES

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ABSTRACT

Sequential A/B testing is a statistical framework used to compare two competing options as data are collected over time. Unlike traditional fixed-sample testing, it is possible to analyse data continuously as data accumulate. During data collection, the available evidence is evaluated to make a decision on whether the two competing options differ or whether additional data should be collected. Because the decision is based on randomly observed data, a formal statistical hypothesis testing framework is required. This decision procedure can be formulated as a sequential hypothesis testing problem.
In this study, two sequential decision approaches, sequential p-value and sequential e-value, are compared for sequential A/B testing under continuous monitoring. The considered framework assumes that observations arrive sequentially and the corresponding outcomes are binary. The performances of these two decision approaches are compared through Monte Carlo simulations under various scenarios.

***K*eywords** Sequential A/B testing · Continuous monitoring · Sequential hypothesis testing · Sequential p-value · Sequential e-value

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ON FIBONACCI–BERNSTEIN BASIS FUNCTIONS AND THEIR PROPERTIES

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ABSTRACT

In this talk, we introduce the Fibonacci–Bernstein basis functions, a Fibonacci analogue of the classical Bernstein basis [1] built from the Fibonomial coefficients and Golden binomial powers of the Golden (Binet–Fibonacci) calculus [2]; they are closely related to the recently introduced F-Bernstein polynomials [3]. For these basis functions we derive an explicit expansion, a generating function, a Golden derivative formula, and a recurrence relation, and we express the Golden hyperbolic functions of both kinds in terms of them. We then introduce the associated Fibonacci–Bernstein matrix, obtain its factorization through the Fibo–Pascal matrices, and determine its inverse with the help of an auxiliary family of polynomials. Finally, we examine the zeros of these basis functions for a specific degree and plot them.

***K*eywords** Fibonacci–Bernstein basis functions · Golden calculus · Fibonomial coefficients · Golden derivative · Fibonacci–Bernstein matrix

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TOWARD LOCALIZED DENGUE CONTROL: UNPACKING CLIMATE DETERMINANTS IN PALU CITY, INDONESIA

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ABSTRACT

Dengue has shifted from a seasonal nuisance to a persistent global health threat, exacerbated by climate change. Since the life cycle of Aedes mosquitoes is strongly controlled by environmental conditions, climate variability offers a crucial early warning signal to anticipate outbreaks. This is particularly relevant for tropical regions like Palu City, Indonesia, which possesses an ideal climate for vector survival. Nevertheless, translating complex climate data into proactive public health actions remains a challenge, as "one-size-fits-all" management policies often fail to accommodate local climate dynamics. This study aims to develop and compare machine learning architectures to accurately predict Dengue Hemorrhagic Fever (DHF) cases in Palu City using climate variables and historical case data. The modeling was evaluated through two approaches: (1) a Random Forest (RF) model, and (2) a hybrid model integrating a linear Autoregressive (AR) algorithm with RF. The AR component captures historical case patterns, while the RF algorithm models the non-linear behavior of climate predictors. The hybrid approach substantially outperformed the single RF model, reducing the Root Mean Square Error (RMSE) by 19.45% and Mean Absolute Error (MAE) by 19.14%. Feature Importance analysis revealed that 1-month lag temperature is a crucial predictor of dengue risk (53.77%), followed by the seasonal effect of the Month variable (25.38%) and 2-month lag relative humidity (20.86%). The model successfully identified critical climate thresholds, indicating that DHF transmission risk increases when the average temperature in the preceding month crosses 26.8°C and humidity two months prior exceeds 87%, particularly during the first-quarter seasonal transition. In conclusion, separating historical components and non-linear climate influences into a combined architecture yields more robust and precise predictions. This model offers a reliable scientific instrument for formulating locally specific vector control policies and supporting a more adaptive DHF early warning system in Palu City.

Keywords Dengue · Random Forest · Autoregressive · Early Warning System · Climate · Palu City.

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AN EFFECTIVE FINITE VOLUME METHOD FOR NONLINEAR MARSHAK WAVE PROBLEMS

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ABSTRACT

In high-energy-density physics, from inertial confinement fusion to astrophysical plasmas, heat is carried primarily by radiation rather than by material conduction. The photon absorption cross-section decreases sharply with temperature, allowing radiation to diffuse much more readily through hot material than through cold material [1]. Under the equilibrium-diffusion approximation, the coupled radiation-diffusion system reduces to a single nonlinear diffusion equation whose temperature-dependent conductivity degenerates as the medium cools. The resulting equation is degenerate parabolic, leading to finite-speed propagation and the formation of a sharp radiation front, in contrast to the instantaneous spreading characteristic of linear diffusion. The Marshak wave problem, in which a radiation source at the boundary drives such a front into a cold medium, provides a classical benchmark for this behavior [2, 3]. Accurately resolving this front is computationally challenging. The strong nonlinearity and degeneracy motivate the use of implicit schemes, which, although unconditionally stable, require solving a large nonlinear algebraic system at every time step. Consequently, the overall computational cost depends strongly on the efficiency of the nonlinear solution procedure [4, 5]. To address this challenge, we develop a conservative finite volume framework coupled with a Jacobian-Free Newton-Krylov (JFNK) method for solving the resulting nonlinear systems. In this context, accurate and computationally efficient numerical methods are essential for reliably resolving the underlying physics and for providing a foundation for more complex radiation-diffusion simulations. The proposed approach is evaluated through mesh and time-step refinement studies, with particular emphasis on solution accuracy, convergence behavior, and computational cost, aiming to establish an effective balance between numerical accuracy and computational efficiency.

***K*eywords** Marshak waves · Nonlinear radiation diffusion · Finite volume method · JFNK method

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SCIV-Weight: A Novel Rank-Correlation-Based Objective Weighting Method Using Importance Vectors for MCDM

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ABSTRACT

Determining criterion weights is a fundamental stage in Multi-Criteria Decision-Making (MCDM) problems, as the assigned weights directly influence the final ranking of alternatives. Existing objective weighting methods generally rely on different characteristics of decision data, such as criterion variability, information content, inter-criteria correlations, or the effects of criteria on the decision matrix. Although approaches such as CRITIC, Entropy, and MEREC are widely used in this field, there remains a need for alternative approaches that place the rank-based relationship between individual criteria and the overall importance structure of alternatives at the center of the weighting mechanism.

This study proposes a novel objective criterion weighting method, termed *SCIV-Weight (Spearman Correlation and Importance Vector-based Weighting)*. The proposed method is based on the Spearman rank correlation between each criterion and a *leave-one-out Importance Vector (IV)* constructed using the remaining criteria. This approach aims to reduce the self-correlation effect that may arise from the direct contribution of the evaluated criterion to the construction of its importance vector and to provide a more representative assessment of the relationship between each criterion and the overall performance structure of alternatives.

The performance of the proposed method is evaluated through a numerical example and comparative analyses with established objective criterion weighting methods. Furthermore, the resulting weights are integrated with the TOPSIS method to examine their effects on the ranking of alternatives. The study aims to introduce a novel objective weighting perspective to the criterion weighting literature by integrating rank-based relationships with an importance-vector-based approach.

***K*eywords:** MCDM, Objective Criteria Weighting, SCIV-Weight

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PRIOR CHOICES AND E-PROCESSES IN SEQUENTIAL A/B TESTING UNDER COMPOSITE NULL HYPOTHESES

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ABSTRACT

In sequential A/B testing, the comparison problem is formulated as a hypothesis test. However, monitoring data continuously challenges classical frameworks and leads to a critical problem: severe Type I error inflation. Moreover, real-world tests involve composite null hypotheses where the true parameter is a range instead of a fixed point. Together, continuous monitoring and this composite structure break traditional likelihood ratio tests and make controlling Type I error difficult.

To resolve these challenges, we apply Bayesian marginalization over the composite null space to construct e-values that accumulate into e-processes. Grounded in martingale theory and Ville's inequality, this framework provides safe evidence for decision-making and guarantees strict Type I error control under continuous monitoring.

Practical performance of this framework is illustrated through a simulation study comparing different prior choices for Bayesian marginalization. The results are then analyzed in terms of Type I error control and decision speed.

***K*eywords** Sequential A/B Testing · E-Processes / E-Values · Martingale · Bayesian Marginalization

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APPARATUS-BASED MATHEMATICS PROBLEMS: CONCEPTUALIZATION AND EARLY EVIDENCE

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ABSTRACT

Based on the observation that problems involving concrete objects have received insufficient attention and that the role of manipulatives in mathematical problem solving has been studied only to a very limited extent, [6] introduced a new type of problem: "Apparatus-Based Mathematics Problems (AbMP)". AbMP is the name given to each problem in a series defined on an apparatus (a group of objects). In other words, AbMP serves both as the name for each individual example within a problem family and as the umbrella term for the family itself. The primary goal of AbMPs is to support the development of mathematical thinking through problem-solving and problem-posing, thereby fostering hand-eye-mind coordination. This study aims to introduce the concept of AbMPs and provide an in-depth overview of graduate theses and conference papers on this topic. Designed as qualitative document analysis, the study employed descriptive content analysis [3] of studies on AbMPs. Within this scope, documents from 5 master's theses, 3 conference papers, and 3 ongoing doctoral dissertations were analyzed. The studies were examined under the headings "research objective," "participant characteristics," "research method," "data collection tools," "types of AbMPs used," and "findings." The study aims to offer insight into research trends and preliminary findings regarding AbMPs. In the graduate theses examined, the AbMPs were used to investigate the problem-solving and problem-posing processes of mathematically gifted students [1], to identify the analytical thinking and reasoning processes of preservice mathematics teacher [5], in the assessment of collaborative problemsolving skills among high school students and preservice teachers [2, 7], and in the analysis of problem-posing experiences of architecture students [4]. It was found that mathematically gifted middle and high school students were able to demonstrate most of the criteria for mathematical giftedness in their AbMP-solving and problem-posing processes and expressed positive views of these problems; that "analytical thinking" patterns were predominant in preservice mathematics teachers' AbMP-solving processes; in terms of reasoning, participants demonstrated logical, advanced visuospatial, and advanced mathematical reasoning; that AbMPs were effective and useful tools in examining preservice teachers' "collaborative problem-solving" skills; the findings showed that architecture students' experiences and products in the processes of solving and designing AbMPs were evaluated as challenging yet distinctive and beneficial, and that functional frameworks for AbMPs could be developed. In conclusion, the findings of the reviewed studies indicate that AbMPs are not merely an alternative problem type across diverse participant groups; rather, they can also serve as valuable research tools that provide insight into problem-solving, problem-posing, reasoning, and higher-order mathematical thinking.

***K*eywords** Apparatus based mathematics problems · Problem solving · Document analysis · Regular solid spherical harmonics

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UNLIMITED SINGLE ITEM LOT SIZING PROBLEM: APPROXIMATION PROCEDURES THROUGH DIMENSIONAL ANALYSIS

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ABSTRACT

The Unlimited Single Item Lot Sizing Problem problem, alias Economic Lot Size (ELS), is often used as sub-problem of more complex Optimization problems because it allows very fast solution approaches. In addition, the ELS mathematical substructure can be find in many real world problems, either in a deterministic mode —e.g., in batch scheduling, capacity expansion, facility location problems—, or in stochastic approaches to production-distribution networks with inventory considerations, prices evolution, and treatment of unsatisfied demands. An extensive study of unlimited single item ELS instances is carried out, and optimal estimates for the ELS problem are treated through dimensionless ratios and convenient quotients: in one hand, by correlating the objective function 's optimal value and the instance parameters; in the other hand, through approximation procedures that first estimate the optimal binary variables and, thereafter, the production and stocking levels. The related results are presented, namely, in what regards the application of dimensional analysis to the ELS instances at hand; key points are also analyzed, such as the better modes to constrain the solutions space, either applying specific cuts, or supported on specific structures for the optimal solution. Finally, further developments are outlined and discussed, including the potential extension of dimensional analysis and associated approximation procedures to other Optimization problems, as well as to real world industry contexts.

***K*eywords** Unlimited Single Item Lot Sizing · Dimensional analysis · Objective function 's optimal value · Optimal binary solution · Approximation procedures

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AN INSIGHT ON FOUNDATION MODELS FOR THE PROCESS INDUSTRY

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ABSTRACT

Foundation models (FM) are important for the process industry because they can help connect data, models, and decisions across complex production systems. The process industry includes chemical, pharmaceutical, food, energy, wood-based, agro-industrial, and circular bio-based industries (CBI) and sectors. These sectors typically operate with strict quality requirements, high capital costs, safety constraints, sustainability targets, and uncertain supply and demand. While traditional digital tools often solve isolated tasks, such as short-term scheduling, maintenance planning, or supply chain optimization, FMs can add value because they can be trained on large and diverse data sources and then adapted to several decision-support tasks [1].

In process industry, useful data are often distributed across sensors, maintenance records, production schedules, or supply chain databases, and FMs can support the integration of such distinct and heterogeneous sources. This is especially relevant for batch scheduling (Batch), where production campaigns, cleaning operations and associated times, storage upper-limits, and delivery dates are strongly connected in order to maximize the equipment utilization. Research on batch processes shows that these scheduling problems can be computationally difficult, which supports the use of advanced computational and heuristic methods [3].

FMs can also support long-range planning (Plan), which usually focus the strategic decisions on capacity expansions of continuous processes. In Plan problems, equipment investment, production targets, inventories, suppliers, and sustainability policies define the limits within which daily schedules must operate. If the Plan macro-level considerations are not aligned with the detailed scheduling constraints, production targets may become infeasible. Optimization-based decision making provides a structured basis for analyzing these links between objectives, constraints, and uncertainty treatment [2]. FMs can complement the widely known Robust Optimization approaches by improving access to data, generating scenarios, or even explaining results trajectories.

The FMs importance is also clear in supply chains (SC). Pharmaceutical SC and other process industries face multi-level risks that often drive to medicines shortages; for instance, on demands variation due to virus outbreaks, disruption on the supplies of APIs or ancillary materials, or even quality failures on manufacturing. Decision support systems and SC optimization methods are needed to improve resilience and reduce shortages [4], [5]. FMs can also assist by organizing knowledge from multiple sources, supporting health emergency systems, and helping to assess operational and supply risks.

However, FMs should not be treated as autonomous authorities in safety-critical industrial decisions, either in Pharmaceutical SC or in energy processing sites. Their outputs must be checked against validated models, processes and equipment constraints, as well as regulatory requirements. Their main value is to support decision making by combining industrial data in real time, optimization methods, and advanced decision support, in a way that can improve long range planning, batch scheduling, and sustainability management in the process industry.

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***K*eywords** Foundation Models · Process Industry · Batch Scheduling · Long range planning

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A UNIFIED NUMBER SYSTEM FOR QUATERNIONS AND HYBRID NUMBERS

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ABSTRACT

In recent years, three distinct number systems : complex ($i^2 = -1$), dual ($\epsilon^2 = 0$), and hyperbolic ($h^2 = 1$) has been studied by many researchers. In our study, we extend this perspective by incorporating Quaternions and Hybrid numbers into a single unified system and investigate its algebraic structure alongside its relevant geometric reflections. The resulting framework provide a higher level of generalization, opening the way for valuable properties and applications.

***K*eywords** Quaternion Numbers · Hybrid Numbers · Matrix Representation.

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POLYNOMIAL RINGS AND FINITE FIELDS: APPLICATIONS TO CYCLIC CODES OVER SPECIAL RINGS

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ABSTRACT

This paper explores the algebraic properties of polynomial rings and finite fields, focusing on their applications to the construction and analysis of cyclic codes defined over special rings (such as finite chain rings and Galois rings). We analyze the algebraic structure of cyclic codes by identifying them as ideals in polynomial quotient rings, facilitating the determination of generator and parity-check polynomials. Furthermore, key properties including minimum distance, duality, and decoding algorithms are examined, highlighting the structural advantages offered by special rings compared to classical finite fields in coding theory and efficient data transmission.

Keywords: polynomial rings, finite fields, special rings, cyclic codes, generator polynomials, Galois rings, finite chain rings.

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YOGITA MAHATEKAR, (INDIA), ORAL PRESENTATION

SOLEY ERSOY, (TÜRKİYE), ORAL PRESENTATION

MURAT BODUR, (TÜRKİYE), ORAL PRESENTATION

QIU-YI CHEN, (CHINA), ORAL PRESENTATION

BISWAJIT PRUSTY, (INDIA), ORAL PRESENTATION

IGNACIO FIDALGO, (SPAIN), ORAL PRESENTATION

MAHPEYKER OZTURK, (TÜRKİYE), ORAL PRESENTATION

RONAN DUPONT, (JAPAN), ORAL PRESENTATION

HANIFA HANIF, (SOUTH AFRICA), ORAL PRESENTATION

YAOWALUCK KHONGTHAM, (THAILAND), ORAL PRESENTATION

LATIFAT ADEBISI ABIMBOLA, (NIGERIA), ORAL PRESENTATION

AVNI JAIN, (INDIA), ORAL PRESENTATION

MOTOHIRO OTSUKA, (JAPAN), ORAL PRESENTATION

MEHMET GÜMÜŞ, (TÜRKİYE), ORAL PRESENTATION

KAZEEM BABAWALE KASALI, (NIGERIA), ORAL PRESENTATION

TIANYUAN ZHAO, (JAPAN), ORAL PRESENTATION

MAHARANI ABU BAKAR, (MALAYSIA), ORAL PRESENTATION

CHATCHAWAN PANRAKSA, (THAILAND), ORAL PRESENTATION

FRANKA MIRIAM BRUECKLER, (CROATIA), ORAL PRESENTATION

CHARITY ALHASSAN, (NIGERIA), ORAL PRESENTATION

ÁLVARO ANTÓN-SANCHO, (SPAIN), ORAL PRESENTATION

ILY AMALINA AHMAD SABRI, (MALAYSIA), ORAL PRESENTATION

CIHAN BAYINDIR, (TÜRKİYE), ORAL PRESENTATION

ELMIRA YU. KALIMULINA, (RUSSIA), ORAL PRESENTATION

SAMET ARPACI, (TÜRKİYE), ORAL PRESENTATION

MUSTAPHA RACHIDI, (BRAZIL), ORAL PRESENTATION

OMER MOHAMED, (KENYA), ORAL PRESENTATION

BAHADDIN SINSOYSAL, (TÜRKİYE), ORAL PRESENTATION

SATI YILMAZ, (TÜRKİYE), ORAL PRESENTATION

KASTRIOT ZOTO, (ALBANIA), ORAL PRESENTATION

ANTON A. ESIN, (RUSSIA), ORAL PRESENTATION

NUR IZZATI KHAIRUDIN, (MALAYSIA), ORAL PRESENTATION

CRISTINA CARIDADE, (PORTUGAL), ORAL PRESENTATION

CAIRES ALBERTO SASSUPE, (ANGOLA), ORAL PRESENTATION

SOUKAINA HADIRI, (MOROCCO), ORAL PRESENTATION

RAQUEL CABALLERO-ÁGUILA, (SPAIN), ORAL PRESENTATION

PAULO SARAIVA, (PORTUGAL), ORAL PRESENTATION

MARIA M. NASCIMENTO, (PORTUGAL), ORAL PRESENTATION

MERVE USEN DOĞDU, (TÜRKİYE), ORAL PRESENTATION

MERVE KARAOĞLAN, (TÜRKİYE), ORAL PRESENTATION

SYERRINA ZAKARIA, (MALAYSIA), ORAL PRESENTATION

ELEN VIVIANI PEREIRA SPREAFICO, (BRAZIL), ORAL PRESENTATION

VERA EGOROVA, (SPAIN), ORAL PRESENTATION

OKTAY PASHAEV, (TÜRKİYE), ORAL PRESENTATION

ESRA KAYA, (TÜRKİYE), ORAL PRESENTATION

ION MIERLUS-MAZILU, (ROMANIA), ORAL PRESENTATION

MURAT BABAARSLAN, (TÜRKİYE), ORAL PRESENTATION

EVGENIY MISHCHENKO, (JAPAN), ORAL PRESENTATION

VIVIANA CELIS, (CHILE), ORAL PRESENTATION

REGINA DE ALMEIDA, (PORTUGAL), ORAL PRESENTATION

NARCISA TEODORESCU, (ROMANIA), ORAL PRESENTATION

MARIUS-ALIN DRAGU, (ROMANIA), ORAL PRESENTATION

ANA PAULA TEIXEIRA, (PORTUGAL), ORAL PRESENTATION

ALI RIZA ALAN, (TÜRKİYE), ORAL PRESENTATION

NAZLIHAN TERZIOĞLU, (TÜRKİYE), ORAL PRESENTATION

DANIELA RICHTARIKOVA, (SLOVAKIA), ORAL PRESENTATION

SANDRA RICARDO, (PORTUGAL), ORAL PRESENTATION

ALICE DAGMAR ANGHELESCU, (ROMANIA), ORAL PRESENTATION

TAKAO KOMATSU, (CHINA), ORAL PRESENTATION

NERIMAN KARTAL, (TÜRKİYE), ORAL PRESENTATION

TARIK GÜL, (TÜRKİYE), ORAL PRESENTATION

ADEM KIZILTEPE, (TÜRKİYE), ORAL PRESENTATION

SALAME KHADIME, (SENEGAL), ORAL PRESENTATION

EMAN KABBAS, (SAUDI ARABIA), ORAL PRESENTATION

FRANCISCO JAVIER REYES BAHAMON, (COLOMBIA), ORAL PRESENTATION

ARACELI QUEIRUGA-DIOS, (SPAIN), ORAL PRESENTATION

FUNDA TAŞDEMIR, (TÜRKİYE), ORAL PRESENTATION

ENGIN KAYNAR, (TÜRKİYE), ORAL PRESENTATION

SEMRA KUŞ, (TÜRKİYE), ORAL PRESENTATION

RALUCA GABRIELA TURTOI, (ROMANIA), ORAL PRESENTATION

CELIL NEBIYEV, (TÜRKİYE), ORAL PRESENTATION

FATMA YEŞIL BARAN, (TÜRKİYE), ORAL PRESENTATION

ANGELLA CONSTANZA ARCE CÁRDENAS, (CHILE), ORAL PRESENTATION

SEHER KÖSE, (TÜRKİYE), ORAL PRESENTATION

MUCAHIT BUYANKARA, (TÜRKİYE), ORAL PRESENTATION

MURAT ÇAĞLAR, (TÜRKİYE), ORAL PRESENTATION

BOUBACAR OUSSEYNOU FALL, (TÜRKİYE), ORAL PRESENTATION

ALEJANDRO MAXIMILIANO ROJAS PALMA, (CHILE), ORAL PRESENTATION

AYSUN ERDOĞAN SERT, (TÜRKİYE), ORAL PRESENTATION

REN GUO, (USA), ORAL PRESENTATION

AYNUR KESKIN KAYMAKCI, (TÜRKİYE), ORAL PRESENTATION

SUMAIYA PARVEEN, (INDIA), ORAL PRESENTATION

FATMA AYBIKE ÇUHA, (TÜRKİYE), ORAL PRESENTATION

AHMET ÖTELEŞ, (TÜRKİYE), ORAL PRESENTATION

ASLI BEKTAŞ KAMIŞLIK, (TÜRKİYE), ORAL PRESENTATION

ANIRUDHA PORIA, (CHINA), ORAL PRESENTATION

DEOLINDA MARIA LOPES DIAS RASTEIRO, (PORTUGAL), ORAL PRESENTATION

FATMA HILAL GÜMÜŞ, (TÜRKİYE), LISTENER

MÜSLÜM GÜMÜŞ, (TÜRKİYE), ORAL PRESENTATION

MEHMET EMRE ÇETIN, (TÜRKİYE), ORAL PRESENTATION

YASIN ÜNSAL, (TÜRKİYE), ORAL PRESENTATION

NUNING NURAINI, (INDONESIA), ORAL PRESENTATION

ECE HAZAL KORKMAZ, (TÜRKİYE), ORAL PRESENTATION

SERKAN AKOGUL, (TÜRKİYE), ORAL PRESENTATION

CIGDEM TOPCU GULOKSUZ, (TÜRKİYE), ORAL PRESENTATION

İREM COŞKUN, (TÜRKİYE), ORAL PRESENTATION

JOAO MIRANDA, (PORTUGAL), ORAL PRESENTATION

YUTO MIYATAKE, (JAPAN), LISTENER

BENDADOU ABDERRAHIM, (ALGERIA), ORAL PRESENTATION

BIANCA LIANA STRATON-BERCEA, (ROMANIA), ORAL PRESENTATION